Zbirka 1.1

Tuesday, March 1, 2022 2:04 PM

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МИЛАН ТАДИЋ, РИФАТ РАМОВИЋ
ЗБИРКА ЗАДАТАКА ИЗ
МИКРОЕЛЕКТРОНИКЕ
АКАДЕМСКА МИСАО

user: kolby
pass: um_08_fe

ДИФУЗИЈА

$$\Phi = -D\nabla N \qquad 1.\Phi.3.$$

$$\frac{\partial N}{\partial t} = \nabla\left(D\nabla N\right) \qquad 2.\Phi.3.$$

$$[N] = \frac{1}{cm^3} \qquad [\Phi] = \frac{1}{cm^2 s} \qquad [D] = \frac{cm^2}{s}$$

$$\Phi = -D\frac{\partial N}{\partial x} \qquad \Phi = -D\frac{\partial N}{\partial x}$$

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial N}{\partial x}\right) \qquad \boxed{\frac{\partial N}{\partial t} = D\frac{\partial^2 N}{\partial x^2}}$$

1) ГАУСОВА ДИФУЗИЈА (ИЗ ОГРАНИЧЕНОГ ИЗВОРА)

$$N(x,t) = \frac{Q}{\sqrt{\pi D t}}\, e^{-\frac{x^2}{4Dt}} \qquad N(0,t) = \frac{Q}{\sqrt{\pi D t}}$$

$$\int_0^{+\infty} N(x,t)\,dx = \frac{Q}{\sqrt{\pi D t}}\underbrace{\int_0^{+\infty} e^{-\frac{x^2}{4Dt}}\,dx} = \frac{Q}{\sqrt{\pi D t}}\,\frac{1}{2}\sqrt{4Dt}\,\sqrt{\pi} = Q$$

$$\int_0^{+\infty} e^{-u^2}\,du = \frac{\sqrt{\pi}}{2}$$

2) erfc ДИФУЗИЈА (ДИФУЗИЈА ИЗ НЕОГРАНИЧЕНОГ ИЗВОРА)

$$N(x,t) = N_0\,\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

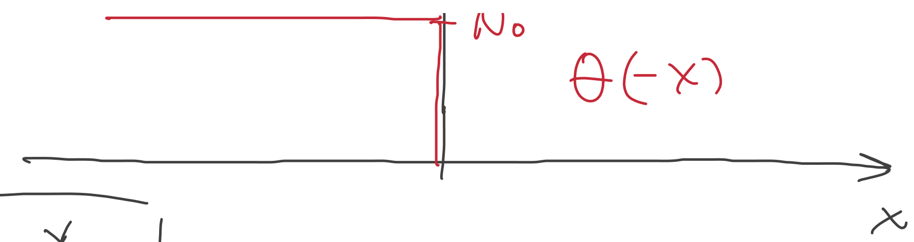
$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^{+\infty} e^{-u^2}\,du$$

$$L = \sqrt{Dt} \quad \text{ДИФУЗИОНА ДУЖИНА}$$

1.1.

$N(x)$

$$N(x,0) = N_0 \theta(-x)$$



$$N(x,t) = ?$$

$$D$$

$$\frac{1}{D}\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial x^2} \qquad \boxed{z = \frac{x}{2\sqrt{Dt}}}$$

$$\frac{\partial N}{\partial t} = \frac{dN}{dz}\frac{\partial z}{\partial t} = N_z' \frac{x}{2\sqrt{D}}\left(-\frac{1}{2}\right)t^{-\frac{3}{2}} = -N_z' \frac{x}{2\sqrt{Dt}}\frac{1}{2t} = -N_z' \frac{z}{2t}$$

$$\frac{\partial N}{\partial x} = \frac{dN}{dz}\frac{\partial z}{\partial x} = N_z' \frac{1}{2\sqrt{Dt}}$$

$$\frac{\partial^2 N}{\partial x^2} = \frac{d}{dz}\left(N_z' \frac{1}{2\sqrt{Dt}}\right)\frac{\partial z}{\partial x} = N_z'' \frac{1}{2\sqrt{Dt}}\frac{1}{2\sqrt{Dt}} = N_z'' \frac{1}{4Dt}$$

$$-\frac{1}{\cancel{D}} N_z' \frac{z}{\cancel{2t}} = N_z'' \frac{1}{\cancel{4Dt}\, 2}$$

$$\boxed{N_z'' = -2zN_z'}$$

$$\frac{dN_z'}{dz} = -2zN_z' \qquad \frac{dN}{dz} = Ce^{-z^2}$$

$$\frac{dN_z'}{N_z'} = -2z\,dz \quad / \int \qquad \int_z^{+\infty} dN = \int_z^{+\infty} C e^{-z^2} dz$$

$$\ln N_z' = -2\frac{z^2}{2} + \ln C \qquad N(\cancel{+\infty}) - N(z) = C\int_z^{+\infty} e^{-z^2} dz$$

$$\ln \frac{N'_z}{C} = -z^2$$

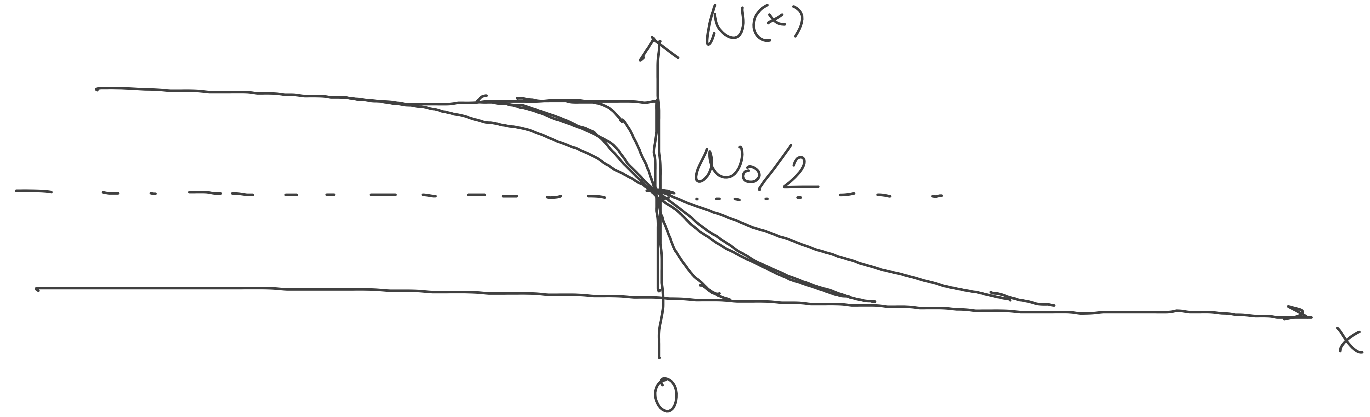
$$\boxed{N(z) = -C\int_{z}^{+\infty} e^{-z^2}\,dz}$$

$$N'_z = C e^{-z^2}$$

$$\boxed{N(-\infty) = N_0} = -C\int_{-\infty}^{+\infty} e^{-z^2}\,dz = -C\sqrt{\pi} \Rightarrow C = -\frac{N_0}{\sqrt{\pi}}$$

$$N(z) = \frac{N_0}{\sqrt{\pi}}\int_{z}^{+\infty} e^{-z^2}\,dz = \frac{N_0}{2}\,\mathrm{erfc}(z) = \boxed{\frac{N_0}{2}\,\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)}$$

$$\mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_{z}^{+\infty} e^{-u^2}\,du$$



Zbirka 1.2

Wednesday, March 2, 2022 9:54 AM

$$N_0 = N(0,t) = \text{const} \qquad N(x,t)$$

$$\frac{1}{D}\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial x^2} \qquad z = \frac{x}{2\sqrt{Dt}} \qquad N_z'' = -2zN_z'$$

$$N(+\infty) - N(z) = C\int_z^{+\infty} e^{-z^2}dz \qquad N(z) = -C\int_z^{+\infty} e^{-z^2}dz$$

$$N(0) = -C\underbrace{\int_0^{+\infty} e^{-z^2}dz}_{} = -C\frac{\sqrt{\pi}}{2} = N_0 \Rightarrow C = -\frac{2}{\sqrt{\pi}}N_0$$

$$N(z) = \frac{2}{\sqrt{\pi}}N_0\int_z^{+\infty} e^{-z^2}dz = N_0\,\mathrm{erfc}(z)$$

$$N(x,t) = N_0\,\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$L(x,s) = \mathcal{L}(N(x,t)) = \int_0^{+\infty} N(x,t)e^{-st}dt \qquad s = \sigma + i\omega$$

$$\mathcal{L}(f^{(n)}(t)) = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$n = 1 \qquad \mathcal{L}\left(\frac{\partial N}{\partial t}\right) = sL(x,s) - \underbrace{N(x,0)}_{=0}$$

УЗОРАК ЈЕ БЕСПРИМЕСН[illegible]

$$\frac{1}{D}\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial x^2} \xrightarrow{\mathcal{L}} \frac{1}{D} s L(x,s) = \frac{d^2 L}{dx^2}$$

$$\boxed{\frac{d^2L}{dx^2} - \frac{s}{D} L = 0} \qquad \lambda^2 - \frac{s}{D} = 0 \qquad \lambda_{1/2} = \pm\sqrt{\frac{s}{D}}$$

$$L(x,s) = \underbrace{C_1}_{} e^{-\sqrt{\frac{s}{D}}x} + \cancel{C_2} e^{\sqrt{\frac{s}{D}}x} \qquad C_2 = 0$$

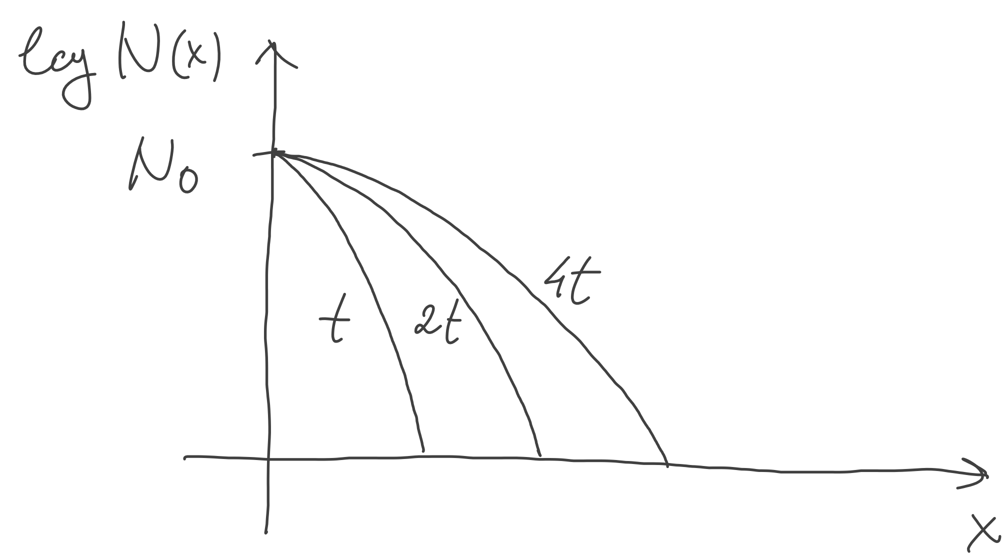
$$C_1 = L(0,s)$$

$$L(0,s) = \mathcal{L}(N(0,t)) = \mathcal{L}(N_0) = \frac{N_0}{s} \qquad \mathcal{L}(1) = \frac{1}{s},\ s > 0$$

$$\boxed{L(x,s) = \frac{N_0}{s} e^{-\sqrt{\frac{s}{D}}x}}$$

$$\mathcal{L}\left[\operatorname{erfc}\left(\frac{k}{2\sqrt{t}}\right)\right] = \frac{1}{s} e^{-k\sqrt{s}} \Rightarrow k = \frac{x}{\sqrt{D}}$$

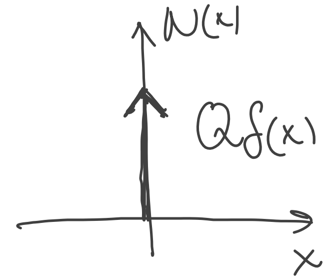
$$N(x,t) = N_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$



Zbirka 1.3

Wednesday, March 2, 2022 10:26 AM

$Q \ , D$ $\qquad \boxed{N(x,0) = Q\delta(x)}$



$$\int_{0^-}^{+\infty} N(x,0)\,dx = \int_{0^-}^{+\infty} Q\,\delta(x) = Q$$

$$\mathcal{C}(f(x)) = F(p) = \int_0^{+\infty} f(x)\cos(px)\,dx \ , \quad 0 < p < +\infty$$

$$\mathcal{C}^{-1}(F(p)) = f(x) = \frac{2}{\pi}\int_0^{+\infty} F(p)\cos(px)\,dp \ , \quad 0 < x < +\infty$$

$$F(p,t) = \int_0^{+\infty} N(x,t)\cos(px)\,dx$$

$$N(x,t) = \frac{2}{\pi}\int_0^{+\infty} F(p,t)\cos(px)\,dp$$

$$\boxed{\mathcal{C}(f''(x)) = -p^2F(p) - f'(0)} \qquad \lim_{x\to+\infty} |f(x)| = 0$$

$$f'(0) = 0$$

$$\frac{\partial^2 N}{\partial x^2} \xrightarrow{\mathcal{C}} -p^2F(p,t)$$

$$\frac{1}{D}\frac{dF}{dt} = -p^2F(p,t) \qquad \boxed{\frac{dF}{dt} = -p^2DF}$$

$$\boxed{F(p,t) = C\,e^{-p^2Dt}}$$

$$C = F(p,0) = \mathcal{C}\left(N(x,0)\right) \qquad \int_{0^-}^{+\infty} \delta(x)\, f(x)\, dx = f(0)$$

$$F(p,0) = \int_0^{+\infty} N(x,0)\cos(px)\,dx = \int_0^{+\infty} Q\,\delta(x)\cos(px)\,dx = Q$$

$$\boxed{F(p,t) = Q e^{-p^2 Dt}}$$

$$N(x,t) = \frac{2}{\pi}\int_0^{+\infty} Q e^{-p^2 Dt}\cos(px)\,dp$$

ПАРНА ФУНКЦИЈА

$$N(x,t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} Q e^{-p^2 Dt}\cos(px)\,dp$$

$$\int_{-\infty}^{+\infty} e^{-p^2 Dt}\, e^{ipx}\,dp = \int_{-\infty}^{+\infty} e^{-p^2 Dt}\left(\cos(px) + i\sin(px)\right)dp$$

$$= \int_{-\infty}^{+\infty} e^{-p^2 Dt}\cos(px)\,dp + i\underbrace{\int_{-\infty}^{+\infty} e^{-p^2 Dt}\sin(px)\,dp}_{0}$$

$$N(x,t) = \frac{Q}{\pi}\int_{-\infty}^{+\infty} e^{-p^2 Dt}\, e^{ipx}\,dp = \frac{Q}{\pi}\int e^{-(p^2 Dt - ipx)}\,dp$$

$-\infty$

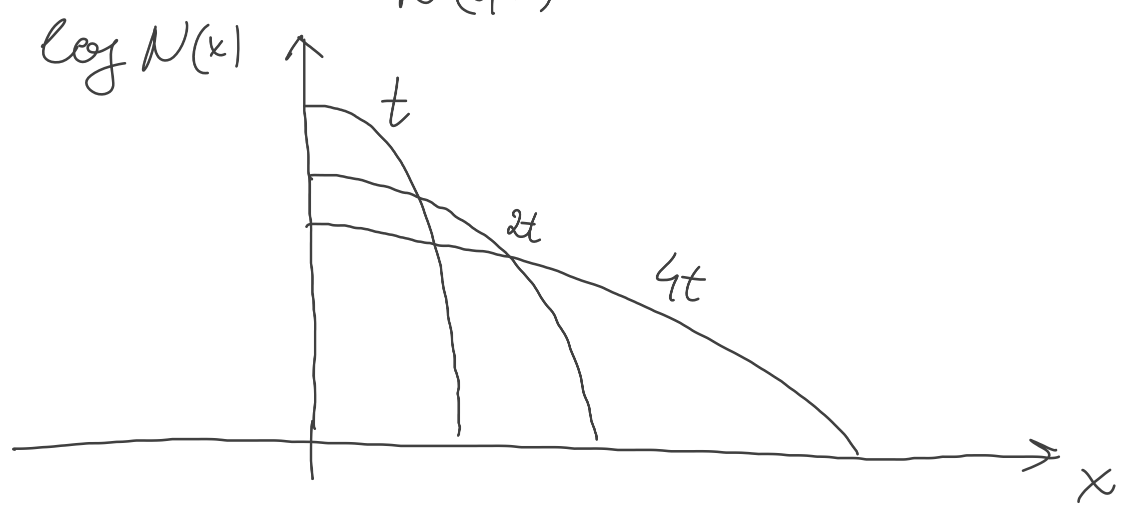
$$-(p^2Dt - ipx) = -\left(\left(\sqrt{Dt}\,p\right)^2 - 2\sqrt{Dt}\,p\,\frac{ix}{2\sqrt{Dt}} - \frac{x^2}{4Dt} + \frac{x^2}{4Dt}\right)$$

$$= -\left(\sqrt{Dt}\,p - \frac{ix}{2\sqrt{Dt}}\right)^2 - \frac{x^2}{4Dt}$$

$$N(x,t) = \frac{Q}{\pi}\int_{-\infty}^{+\infty} e^{-\left(\sqrt{Dt}\,p - \frac{ix}{2\sqrt{Dt}}\right)^2} e^{-\frac{x^2}{4Dt}}\,dp = \frac{Q}{\pi} e^{-\frac{x^2}{4Dt}} \int_{-\infty}^{+\infty} e^{-\left(\sqrt{Dt}\,p - \frac{ix}{2\sqrt{Dt}}\right)^2} dp$$

$$N(x,t) = \frac{Q}{\pi} e^{-\frac{x^2}{4Dt}} \frac{\sqrt{\pi}}{\sqrt{Dt}} = \boxed{\underbrace{\frac{Q}{\sqrt{\pi Dt}}}_{N(0,t)} e^{-\frac{x^2}{4Dt}}}$$

$$\int_{-\infty}^{+\infty} e^{-(ax-b)^2}dx = \frac{\sqrt{\pi}}{a}$$



$$\frac{1}{D}\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial x^2} \xrightarrow{\mathcal{L}} \frac{1}{D}\left(sL(x,s) - \cancelto{0}{N(x,0)}\right) = \frac{d^2L}{dx^2}$$

$$\mathcal{L}(N(x,t)) = \int_0^{+\infty} N(x,t)e^{-st}\,dt = L(x,s)$$

$$\frac{d^2L}{dx^2} - \frac{s}{D}L = 0 \qquad L(x,s) = C_1 e^{-\sqrt{\frac{s}{D}}x} + \underset{0}{\cancel{C_2}} e^{\sqrt{\frac{s}{D}}x}$$

$$Q = \int_0^{+\infty} N(x,t)\,dx \qquad \mathcal{L}\left\{Q = \int_0^{+\infty} N(x,t)\,dx\right\}$$

$$\frac{Q}{s} = \int_0^{+\infty} L(x,s)\,dx = \int_0^{+\infty} C_1 e^{-\sqrt{\frac{s}{D}}x} = C_1\left(-\sqrt{\frac{D}{s}}\right)\left(e^{-\infty} - e^{0}\right)$$

$$\frac{Q}{s} = C_1\sqrt{\frac{D}{s}} \Rightarrow C_1 = \frac{Q}{\sqrt{Ds}}$$

$$L(x,s) = \frac{Q}{\sqrt{Ds}} e^{-\sqrt{\frac{s}{D}}x} = \mathcal{L}\left(N(x,t)\right)$$

$$\mathcal{L}\left(\frac{1}{\sqrt{\pi t}} e^{-\frac{a^2}{4t}}\right) = \frac{1}{\sqrt{s}} e^{-a\sqrt{s}} \Rightarrow a = \frac{x}{\sqrt{D}}$$

$$N(x,t) = \frac{Q}{\sqrt{\pi D t}} e^{-\frac{x^2}{4Dt}}$$

Zbirka 1.7

Wednesday, March 9, 2022 8:39 AM

$x > 0 \qquad N(x) = Q\delta(x)$

$t_0, T_0 \qquad D_0$

$t, T \qquad D$

$$\frac{1}{D}\frac{\partial N}{\partial x} = \frac{\partial^2 N}{\partial x^2} \xrightarrow{\mathcal{C}} \boxed{\frac{1}{D}\frac{dF}{dt} = -p^2 F}$$

$$F(p,t) = \int_0^{+\infty} N(x,t)\cos(px)\,dx$$

$$F(p,t) = Ce^{-p^2 Dt}$$

$$F(p,t_0) = F(p,0)e^{-p^2 D_0 t_0}$$

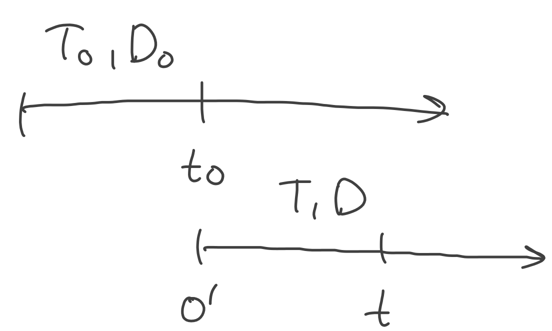
$$F(p,0) = \int_0^{+\infty} N(x,0)\cos(px)\,dx = \int_0^{+\infty} Q\delta(x)\cos(px)\,dx = Q$$

$$F(p,t_0) = Qe^{-p^2 D_0 t_0}$$

$$(1.3.) \rightarrow N(x,t_0) = \frac{2}{\pi}\int_0^{+\infty} F(p,t_0)\cos(px)\,dp$$

$$N(x,t_0) = \frac{Q}{\sqrt{\pi D_0 t_0}}e^{-\frac{x^2}{4D_0 t_0}}$$

$$N(x,0') = \frac{Q}{\sqrt{\pi D_0 t_0}}e^{-\frac{x^2}{4D_0 t_0}}$$



$$\frac{dF}{dt} = -p^2 DF$$

$$\boxed{F(p,0') = Q e^{-p^2 D_0 t_0}}$$

$$F(p,t) = F(p,0') e^{-p^2 Dt} = Q e^{-p^2 D_0 t_0} e^{-p^2 Dt} = Q e^{-p^2 (D_0 t_0 + Dt)}$$

$$N(x,t) = \frac{2}{\pi} \int_0^{+\infty} Q e^{-p^2 (D_0 t_0 + Dt)} \cos(px)\, dp = \frac{Q}{\pi} \int_{-\infty}^{+\infty} e^{-p^2 (D_0 t_0 + Dt)} e^{ipx}\, dp$$

1.3.

$$e^{ipx} = \cos(px) + \underline{i \sin(px)}$$

ДОПУНА ДО ПОТПУНОГ КВАДРАТА

ПОАСОНОВ ИНТЕГРАЛ

$$N(x,t) = \frac{Q}{\sqrt{\pi (D_0 t_0 + Dt)}} e^{-\frac{x^2}{4(D_0 t_0 + Dt)}}$$

Zbirka 1.8

Wednesday, March 9, 2022 8:39 AM

$T_0 \quad t_0 = 10\ \text{min}$

$$T = \frac{T_0}{1+\frac{t}{t_1}}, \quad t < t_1 \qquad T_0 = 1000^\circ C \qquad t_1 = 20\ \text{min}$$

$$T_0 = 1273\,K$$

$$\frac{N(0,t')}{N(0,0)} \qquad T(t') = \frac{T_0}{2} = \frac{T_0}{1+\frac{t'}{t_1}} \Rightarrow t' = t_1$$

$$\frac{N(0,t_1)}{N(0,0)} = ? \qquad D_\infty = 3,85\ \text{cm}^2/\text{s} \qquad E_a = 3,46\,\text{eV}$$

$$N(x,t_0) = \frac{Q}{\sqrt{\pi D_0 t_0}} e^{-\frac{x^2}{4D_0 t_0}} \quad 1.7. \rightarrow N(x,t) = \boxed{\frac{Q}{\sqrt{\pi(D_0 t_0 + Dt)}}} e^{-\frac{x^2}{4(D_0 t_0 + Dt)}}$$

$$D = D_\infty e^{-\frac{E_a}{k_B T}} = D_\infty e^{-\frac{E_a}{k_B \frac{T_0}{1+\frac{t}{t_1}}}} = D_\infty e^{-\frac{E_a\left(1+\frac{t}{t_1}\right)}{k_B T_0}}$$

$$t_1 = N\Delta t \qquad \begin{matrix} D_1, & D_2, & D_3, & D_4, & \dots, & D_N \\ \Delta t, & \Delta t, & \Delta t, & \Delta t, & \dots, & \Delta t \end{matrix}$$

$$N(0,t_1) = \frac{Q}{\sqrt{\pi\left(D_0 t_0 + D_1 \Delta t + D_2 \Delta t + \dots D_N \Delta t\right)}} = \frac{Q}{\sqrt{\pi\left(D_0 t_0 + \sum_{i=1}^{N} D_i \Delta t\right)}}$$

$$\Delta t \to 0 \qquad \sum_{i=1}^{N} D_i \Delta t = \int_0^{t_1} D(t)\,dt \qquad N(0,t_1) = \frac{Q}{\sqrt{\pi D_0 t_0 + \pi \int_0^{t_1} D(t)\,dt}}$$

$$N(0,0) = \frac{Q}{\sqrt{\pi D_0 t_0}}$$

$$\frac{N(0,t_1)}{N(0,0)} = \sqrt{\frac{D_0 t_0}{D_0 t_0 + \int_0^{t_1} D(t)\,dt}} = 0{,}9697 = 0{,}97$$

$$\int_0^{t_1} D(t)\,dt = \int_0^{t_1} D_\infty e^{-\frac{E_a}{k_B T_0}\left(1 + \frac{t}{t_1}\right)} dt = D_\infty e^{-\frac{E_a}{k_B T_0}} \int_0^{t_1} e^{-\frac{E_a}{k_B T_0}\frac{t}{t_1}} dt$$

$$= \underbrace{D_\infty e^{-\frac{E_a}{k_B T_0}}}_{D_0} t_1 \frac{k_B T_0}{E_a}\left(1 - e^{-\frac{E_a}{k_B T_0}}\right) = 2{,}9362 \cdot 10^{-12}\ \mathrm{cm}^2$$

$$\frac{E_a}{k_B T_0} = 31{,}5408 \qquad D_0 = D_\infty e^{-\frac{E_a}{k_B T_0}} = 7{,}7176 \cdot 10^{-14}\ \frac{\mathrm{cm}^2}{\mathrm{s}}$$

Zbirka 1.13

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$Z, n_i \Rightarrow$ ФАКТОР УБРЗАЊА ДИФУЗИЈЕ

$$\Phi = -D \underline{\underline{g}} \frac{dN}{dx} \qquad g = g(Z, n_i, N)$$

$$\Phi = \underbrace{-D\frac{dN}{dx}}_{\text{ДИФУЗИЈА}} + \underbrace{vN}_{\text{ДРИФТ}} \qquad v = \mu \mathcal{E}$$

μ - ПОКРЕТЉИВОСТ

$\mathcal{E}$ - ел. ПОЉЕ

$$\frac{D}{\mu} = \frac{k_B T}{qZ} = \frac{V_t}{Z} \Rightarrow \mu = \frac{DZ}{V_t}$$

$$v = \frac{DZ}{V_t}\mathcal{E}$$

$$\Phi = -D\frac{dN}{dx} + D\frac{Z}{V_t}N\underline{\underline{\mathcal{E}}}$$

$$\mathcal{E} = -\underbrace{\frac{D_n}{\mu_n}}\frac{1}{n}\frac{dn}{dx} = -V_t\frac{1}{n}\frac{dn}{dx}$$

$$J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx} = 0$$

$$n\mu_n\mathcal{E} = -D_n\frac{dn}{dx}$$

$$\Phi = -D\frac{dN}{dx} - DZN\underbrace{\frac{1}{n}\frac{dn}{dx}} \qquad \Phi = -Dg\frac{dN}{dx}$$

$$\frac{dn}{dx} = \underbrace{\frac{dn}{dN}}\frac{dN}{dx}$$

$$\left.\begin{array}{l} p + N = n \\ pn = n_i^2 \end{array}\right\} \quad \frac{n_i^2}{n} + N = n$$

$$n^2 - nN - n_i^2 = 0$$

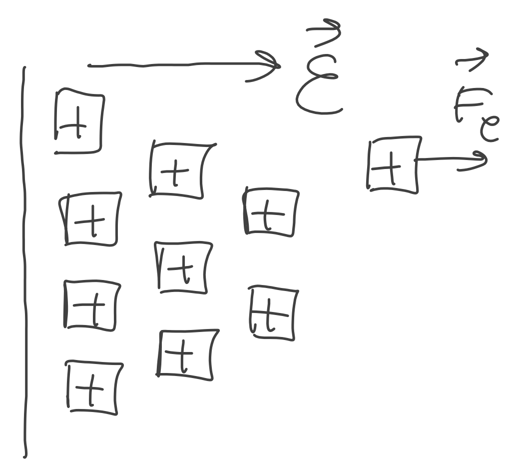
$$n = \frac{N \oplus \sqrt{N^2 + 4n_i^2}}{2} = \frac{N}{2} + \sqrt{\frac{N^2}{4} + n_i^2}$$

$$\frac{dn}{dN} = \frac{1}{2} + \frac{1}{2\sqrt{\frac{N^2}{4} + n_i^2}} \frac{N}{2} = \frac{1}{2} \frac{\frac{N}{2} + \sqrt{\frac{N^2}{4} + n_i^2}}{\sqrt{\frac{N^2}{4} + n_i^2}} = \frac{1}{2} \frac{n}{\sqrt{\frac{N^2}{4} + n_i^2}} = \frac{n}{\sqrt{N^2 + 4n_i^2}}$$

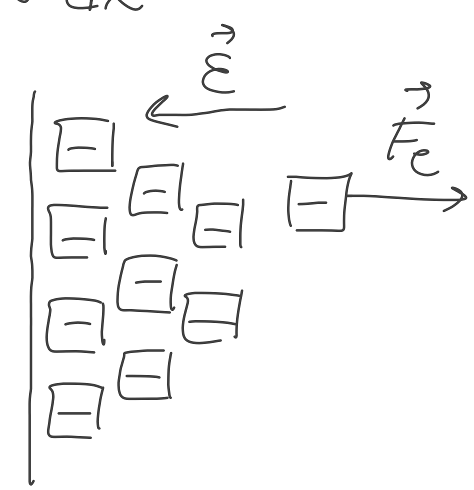
$$\frac{1}{n}\frac{dn}{dx} = \frac{1}{n}\frac{dn}{dN}\frac{dN}{dx} = \frac{1}{n}\frac{n}{\sqrt{N^2 + 4n_i^2}}\frac{dN}{dx} = \frac{1}{\sqrt{N^2 + 4n_i^2}}\frac{dN}{dx}$$

$$\phi = -D\frac{dN}{dx} - DZN\frac{1}{\sqrt{N^2 + 4n_i^2}}\frac{dN}{dx} \qquad \phi = -Dg\frac{dN}{dx}$$

$$\phi = -D\underbrace{\left(1 + \frac{ZN}{\sqrt{N^2 + 4n_i^2}}\right)}_{g}\frac{dN}{dx}$$



ДОНОРИ



АКЦЕПТОРИ

Zbirka 2.3

Wednesday, March 9, 2022 8:39 AM

$$N(x) = \frac{Q}{\sqrt{2\pi}\,\Delta R_p} e^{-\frac{(x-R_p)^2}{2\Delta R_p^2}}$$

$$Q = \int_{-\infty}^{+\infty} N(x)\,dx$$ УКУПНА ДОЗА $\qquad \bar{N} = QA \qquad$ A – ПОВРШИНА

$$R_p = \bar{x} = \frac{1}{Q}\int_{-\infty}^{+\infty} x N(x)\,dx$$ ПРОЈЕКТОВАНИ ДОМЕТ

$$\Delta R_p^2 = \frac{1}{Q}\int_{-\infty}^{+\infty} (x-R_p)^2 N(x)\,dx$$

$D = 125\ mm$ Si

B $E = 80\ keV$

$Q = 3 \cdot 10^{14}\ cm^{-2}$

$N_0 = ?$

$T(x=d) = ?$

$d = 0{,}5\ \mu m$

$t = 1\ min$

$I = ?$

$$N(x) = \underbrace{\frac{Q}{\sqrt{2\pi}\,\Delta R_p}}_{N_0} e^{-\frac{(x-R_p)^2}{2\Delta R_p^2}}$$

$$N_0 = \frac{Q}{\sqrt{2\pi}\,\Delta R_p} = \frac{3\cdot 10^{14}}{\sqrt{2\pi}\cdot 0{,}064\cdot 10^{-4}} = 1{,}87\cdot 10^{19}\ cm^{-3}$$

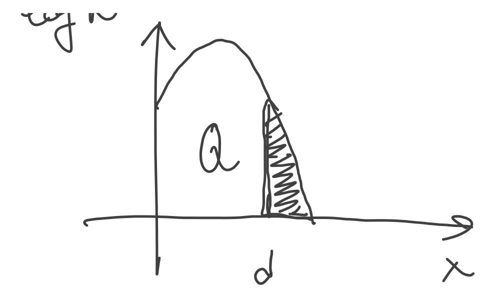
$$I = \frac{q\bar{N}}{t} = \frac{q}{t}\,\frac{Q\pi D^2}{4} = \frac{1{,}6\cdot 10^{-19}}{60}\,\frac{\pi\,(12{,}5)^2\cdot 3\cdot 10^{14}}{4}$$

$$I = 9{,}8175\cdot 10^{-5}\ A = 98\ \mu A$$

$R_p = 0{,}25\ \mu m$

$\Delta R_p = 0{,}064\ \mu m$

$$T = \frac{1}{Q}\int_d^{+\infty} N(x)\,dx$$

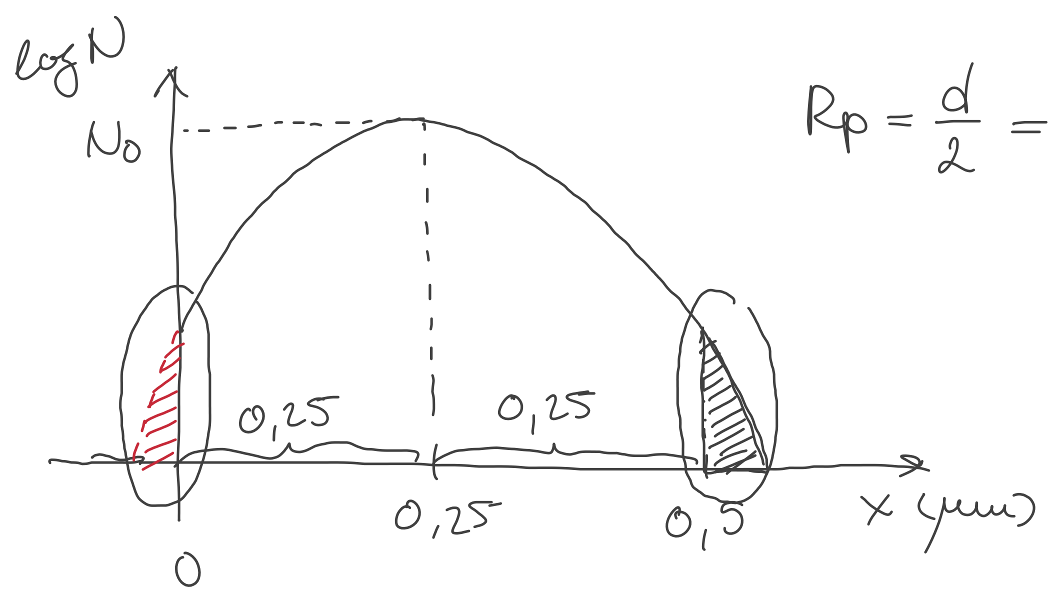


$$T = \frac{1}{\sqrt{2\pi}\Delta R_p}\int_d^{+\infty} e^{-\frac{(x-R_p)^2}{2\Delta R_p^2}}\,dx \qquad \frac{x-R_p}{\sqrt{2}\Delta R_p} = z \qquad \frac{dx}{\sqrt{2}\Delta R_p} = dz$$

$$T = \frac{1}{\cancel{\sqrt{2\pi}}\ \cancel{\Delta R_p}}\cancel{\sqrt{2}\,\Delta R_p}\,\frac{2}{2}\int_{\frac{d-R_p}{\sqrt{2}\Delta R_p}}^{+\infty} e^{-z^2}\,dz = \frac{1}{2}\,\mathrm{erfc}\left(\frac{d-R_p}{\sqrt{2}\Delta R_p}\right) \qquad \mathrm{erfc}(z) = \frac{2}{\sqrt{\pi}}\int_z^{+\infty} e^{-u^2}\,du$$

$$u = \frac{d-R_p}{\sqrt{2}\Delta R_p} = 2{,}7621 \approx 2{,}8 \qquad \mathrm{erfc}(z) \approx \frac{e^{-z^2}}{\sqrt{\pi}\,z}\ ,\ z >> 1$$

$$T \approx \frac{e^{-u^2}}{2\sqrt{\pi}\,u} = 4{,}963\cdot 10^{-5} = \underline{0{,}00496\%}$$



$R_p = \frac{d}{2} = 0{,}25\ \mu m$

Zbirka 2.4

Wednesday, March 9, 2022 8:39 AM

$$N(x) = \begin{cases} N_0 \exp\left[-\dfrac{(x-R_m)^2}{2\Delta R_{p_1}^2}\right], & x < R_m \\ N_0 \exp\left[-\dfrac{(x-R_m)^2}{2\Delta R_{p_2}^2}\right], & x > R_m \end{cases}$$

$N_0,\ R_p,\ \Delta R_p = ?$

$R_m = 68{,}94\ \text{nm}$ $\quad \Delta R_{p_1} = 24{,}70\ \text{nm}$

$\Delta R_{p_2} = 34{,}35\ \text{nm}$

$Q = 10^{15}\ \text{cm}^{-2}$

$$Q = \int_{-\infty}^{+\infty} N(x)\,dx$$

$$Q = N_0 \int_{-\infty}^{R_m} e^{-\frac{(x-R_m)^2}{2\Delta R_{p_1}^2}}\,dx + N_0 \int_{R_m}^{+\infty} e^{-\frac{(x-R_m)^2}{2\Delta R_{p_2}^2}}\,dx$$

$$\frac{x-R_m}{\sqrt{2}\Delta R_{p_1}} = t_1 \qquad \frac{x-R_m}{\sqrt{2}\Delta R_{p_2}} = t_2$$

$$\frac{dx}{\sqrt{2}\Delta R_{p_1}} = dt_1 \qquad \frac{dx}{\sqrt{2}\Delta R_{p_2}} = dt_2$$

$$Q = N_0 \int_{-\infty}^{0} e^{-t_1^2}\,dt_1 \left(\sqrt{2}\Delta R_{p_1}\right) + N_0 \int_{0}^{+\infty} e^{-t_2^2}\,dt_2 \left(\sqrt{2}\Delta R_{p_2}\right)$$

$$Q = N_0 \frac{\sqrt{\pi}}{2}\sqrt{2}\,\Delta R_{p1} + N_0 \frac{\sqrt{\pi}}{2}\sqrt{2}\,\Delta R_{p2} = N_0 \sqrt{\frac{\pi}{2}}\left(\Delta R_{p1} + \Delta R_{p2}\right)$$

$$N_0 = \sqrt{\frac{2}{\pi}}\,\frac{Q}{\Delta R_{p1} + \Delta R_{p2}} = 1{,}3512 \cdot 10^{20}\ \mathrm{cm}^{-3}$$

2.4 - projektovani domet i projektovano rasipanje

Tuesday, March 15, 2022 7:11 PM

$$R_p = \frac{1}{Q}\int_{-\infty}^{+\infty} x\, N(x)\, dx$$

$$R_p = \frac{N_0}{Q}\int_{-\infty}^{R_m} x\, e^{-\frac{(x-R_m)^2}{2\Delta R_{p1}^2}}\, dx + \frac{N_0}{Q}\int_{R_m}^{+\infty} x\, e^{-\frac{(x-R_m)^2}{2\Delta R_{p2}^2}}\, dx$$

$$\frac{(x-R_m)^2}{2\Delta R_{p1}^2} = t_1 \qquad \frac{(x-R_m)^2}{2\Delta R_{p2}^2} = t_2$$

$$\frac{x-R_m}{\Delta R_{p1}^2}\, dx = dt_1 \qquad \frac{x-R_m}{\Delta R_{p2}^2}\, dx = dt_2$$

$$(x-R_m)\, dx = \Delta R_{p1}^2\, dt_1 \qquad (x-R_m)\, dx = \Delta R_{p2}^2\, dt_2$$

$$R_p = \frac{N_0}{Q}\int_{-\infty}^{R_m} (x-R_m)\, e^{-\frac{(x-R_m)^2}{2\Delta R_{p1}^2}}\, dx + \frac{N_0}{Q}\int_{R_m}^{+\infty} (x-R_m)\, e^{-\frac{(x-R_m)^2}{2\Delta R_{p2}^2}}\, dx$$

$$+ \frac{N_0}{Q} R_m \int_{-\infty}^{R_m} e^{-\frac{(x-R_m)^2}{2\Delta R_{p1}^2}}\, dx + \frac{N_0}{Q} R_m \int_{R_m}^{+\infty} e^{-\frac{(x-R_m)^2}{2\Delta R_{p2}^2}}\, dx$$

$$R_p = \frac{N_0}{Q}\Delta R_{p1}^2 \underbrace{\int_{+\infty}^{0} e^{-t_1}\, dt_1}_{-1} + \frac{N_0}{Q}\Delta R_{p2}^2 \underbrace{\int_{0}^{+\infty} e^{-t_2}\, dt_2}_{1} + R_m \underbrace{\frac{1}{Q}\int_{-\infty}^{+\infty} N(x)\, dx}_{=1}$$

$$R_p = \frac{N_0}{Q}\Delta R_{p_1}^2(-1) + \frac{N_0}{Q}\Delta R_{p_2}^2(1) + R_m = R_m + \frac{N_0}{Q}\left(\Delta R_{p_2}^2 - \Delta R_{p_1}^2\right)$$

$$R_p = R_m + \frac{N_0}{Q}\left(\Delta R_{p_2} - \Delta R_{p_1}\right)\left(\Delta R_{p_2} + \Delta R_{p_1}\right)$$

$$R_p = R_m + \sqrt{\frac{2}{\pi}}\,\frac{1}{\cancel{\Delta R_{p_1} + \Delta R_{p_2}}}\left(\Delta R_{p_2} - \Delta R_{p_1}\right)\cancel{\left(\Delta R_{p_2} + \Delta R_{p_1}\right)}$$

$$R_p = R_m + \sqrt{\frac{2}{\pi}}\left(\Delta R_{p_2} - \Delta R_{p_1}\right) = 76{,}64\ \text{nm}$$

$$\Delta R_p^2 = \frac{1}{Q}\int_{-\infty}^{+\infty}(x-R_p)^2 N(x)\,dx$$

$$\Delta R_p^2 = \frac{1}{Q}\int_{-\infty}^{+\infty}x^2 N\,dx - \overbrace{\frac{2R_p}{Q}\int_{-\infty}^{+\infty}xN\,dx}^{=R_p} + \overbrace{\frac{R_p^2}{Q}\int_{-\infty}^{+\infty}N\,dx}^{=1}$$

$$\Delta R_p^2 = \underbrace{\frac{1}{Q}\int_{-\infty}^{+\infty}x^2 N(x)\,dx}_{I_1} - R_p^2$$

$$I_1 = \frac{1}{Q}\int_{-\infty}^{+\infty}(x-R_m)^2 N\,dx + \overbrace{\frac{2R_m}{Q}\int_{-\infty}^{+\infty}xN\,dx}^{=R_p} - \overbrace{\frac{R_m^2}{Q}\int_{-\infty}^{+\infty}N\,dx}^{=1}$$

$$I_1 = \frac{1}{Q}\int^{+\infty}(x-R_m)^2 N\,dx + 2R_m R_p - R_m^2$$

$$\int^{+\infty} x^2 e^{-x^2} \ldots \sqrt{\pi}$$

$$\underbrace{\qquad\qquad -\infty \qquad\qquad}_{I_2} \qquad\qquad \int_0 \sim e \quad dx = \frac{\quad}{4}$$

$$I_2 = \frac{N_0}{Q}\int_{-\infty}^{R_m}(x-R_m)^2 e^{-\frac{(x-R_m)^2}{2\Delta R p_1^2}}dx + \frac{N_0}{Q}\int_{R_m}^{+\infty}(x-R_m)^2 e^{-\frac{(x-R_m)^2}{2\Delta R p_2^2}}dx$$

$$\frac{x-R_m}{\sqrt{2}\Delta R p_{1/2}} = t_{1/2} \qquad (x-R_m)^2 = 2\Delta R p_{1/2} t_{1/2}^2 \qquad dx = \sqrt{2}\Delta R p_{1/2} dt_{1/2}$$

$$I_2 = \frac{N_0}{Q}\int_{-\infty}^{0}(2\Delta R p_1 t_1^2) e^{-t_1^2}(\sqrt{2} R p_1 dt_1) + \frac{N_0}{Q}\int_{0}^{+\infty}(2\Delta R p_2^2 t_2^2) e^{-t_2^2}(\sqrt{2}\Delta R p_2 dt_2)$$

$$= \frac{N_0}{Q} 2\sqrt{2}\Delta R p_1^2 \int_{-\infty}^{0} t_1^2 e^{-t_1^2} dt_1 + \frac{N_0}{Q} 2\sqrt{2}\Delta R p_2^3 \int_0^{+\infty} t_2^2 e^{-t_2^2} dt_2$$

$$I_2 = \frac{N_0}{Q}\cancel{2\sqrt{2}}\frac{\sqrt{\pi}}{\cancel{4}}(\Delta R p_1^3 + \Delta R p_2^3)$$

$$I_2 = \cancel{\sqrt{\frac{2}{\pi}}\frac{1}{\Delta R p_1 + \Delta R p_2}\sqrt{\frac{\pi}{2}}(\Delta R p_1 + \Delta R p_2)}(\Delta R p_1^2 - \Delta R p_1 \Delta R p_2 + \Delta R p_2^2)$$

$$I_1 = I_2 + 2R_m R_p - R_m^2$$

$$\Delta R p^2 = I_1 - R_p^2 \qquad\qquad -(R_p - R_m)^2 \qquad R_p - R_m = \sqrt{\frac{2}{\pi}}(\Delta R p_2 - \Delta R p_1)$$

$$\Delta R p^2 = \Delta R p_1^2 - \Delta R p_1 \Delta R p_2 + \Delta R p_2^2 + \overbrace{2R_m R_p - R_m^2 - R_p^2}^{-(R_p-R_m)^2}$$

I_2

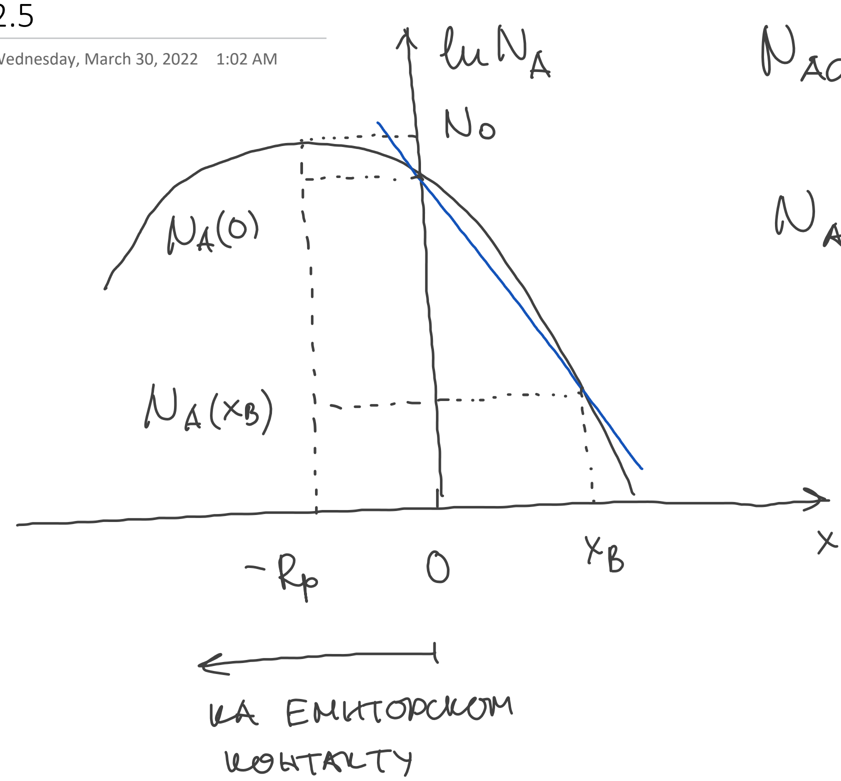
I_1

$$\Delta R_p^2 = \Delta R_{p1}^2 - \Delta R_{p1}\Delta R_{p2} + \Delta R_{p2}^2 - \frac{2}{\pi}\left(\Delta R_{p2} - \Delta R_{p1}\right)^2$$

$$\Delta R_p = 29{,}703 \text{ nm}$$

2.5

Wednesday, March 30, 2022 1:02 AM



$$N_{AG}(x) = N_0 e^{-\frac{(x+R_p)^2}{2\Delta R_p^2}} = N_0 e^{-\frac{x^2+2xR_p+R_p^2}{2\Delta R_p^2}}$$

$$N_{AG}(x) = N_0 e^{-\frac{R_p^2}{2\Delta R_p^2}} e^{-\frac{x(x+2R_p)}{2\Delta R_p^2}} = N_A(0) e^{-\frac{x(x+2R_p)}{2\Delta R_p^2}}$$

$$J_p = q p_B \mu_p \mathcal{E} - q D_p \frac{dp_B}{dx} = 0$$

$$\mathcal{E} = \frac{D_p}{\mu_p}\frac{1}{p_B}\frac{dp_B}{dx} = V_t \frac{1}{p_B}\frac{dp_B}{dx} = V_t \frac{1}{N_A}\frac{dN_A}{dx}$$

$$p_B(x) = N_A(x)$$

$$\langle \mathcal{E}_G \rangle = \langle \mathcal{E}_{exp} \rangle$$

$$\mathcal{E}_G = \frac{V_t}{N_{AG}}\frac{dN_{AG}}{dx} = \frac{V_t}{N_{AG}} N_{AG}\left(-\frac{x+R_p}{\Delta R_p^2}\right) = -V_t \frac{x+R_p}{\Delta R_p^2}$$

$$\langle \mathcal{E}_G \rangle = \frac{1}{x_B}\int_0^{x_B} \mathcal{E}_G \, dx = \frac{1}{x_B}\int_0^{x_B} -V_t \frac{x+R_p}{\Delta R_p^2} dx$$

$$\langle \mathcal{E}_G \rangle = -\frac{V_t}{x_B \Delta R_p^2}\left(\frac{x^2}{2}\Big|_0^{x_B} + R_p x \Big|_0^{x_B}\right) = -\frac{V_t(x_B+2R_p)}{2\Delta R_p^2}$$

$$N_{A\,exp}(x) = N_A(0) e^{-a\frac{x}{x_B}} \qquad a = \ln\left(\frac{N_A(0)}{N_A(x_B)}\right)$$

$$\mathcal{E}_{exp} = \frac{V_t}{N_{A\,exp}}\frac{dN_{A\,exp}}{dx} = -V_t \frac{a}{x_B} = \langle \mathcal{E}_{exp} \rangle$$

$$N_{AG}(x_B) = N_{A\,exp}(x_B)$$

$$\langle \mathcal{E}_{exp} \rangle = -V_t \frac{a}{x_B} = -\frac{V_t(x_B+2R_p)}{2\Delta R_p^2} = \langle \mathcal{E}_G \rangle$$

$x_B(x_B+2R_p)$

$$N_A(0)\, e^{-\frac{x_B(x_B+2R_p)}{2\Delta R_p^2}} = N_A(0)\, e^{-a}$$

$$a = \frac{x_B(x_B+2R_p)}{2\Delta R_p^2}$$

Zbirka 2.6

Wednesday, March 16, 2022 9:48 AM

$$f(x) = \frac{N(x)}{Q} \qquad \int_{-\infty}^{+\infty} f(x)\,dx = \frac{1}{Q}\int_{-\infty}^{+\infty} N(x)\,dx = 1$$

$$m_i = \int_{-\infty}^{+\infty} s^i f(s)\,ds, \quad s = x - R_p, \quad i \geq 0$$ i-ТИ ЦЕНТРАЛНИ МОМЕНТ

$$m_0 = \int_{-\infty}^{+\infty} f(s)\,ds = 1$$

$$m_1 = \int_{-\infty}^{+\infty} s f(s)\,ds = \int_{-\infty}^{+\infty} (x - R_p) f\,ds = \int_{-\infty}^{+\infty} x \frac{N(x)}{Q}\,dx - R_p \int_{-\infty}^{+\infty} f\,ds = R_p - R_p = 0$$

$$m_2 = \int_{-\infty}^{+\infty} (x - R_p)^2 f(s)\,ds = \Delta R_p^2 = \sigma^2$$

$\gamma = \dfrac{m_3}{\Delta R_p^3}$ АСИМЕТРИЈА $\qquad \beta = \dfrac{m_4}{\Delta R_p^4}$ СПЉОШТЕНОСТ

$$\frac{df}{ds} = \frac{s - a}{b_0 + b_1 s + b_2 s^2} f(s) \qquad \sigma, \gamma, \beta \qquad a, b_0, b_1, b_2 = ?$$

$$s f(s) = a f(s) + (b_0 + b_1 s + b_2 s^2)\frac{df}{ds} \quad \Big/ \int_{-\infty}^{+\infty} s^i (\quad)\,ds$$

$$\int_{-\infty}^{+\infty} s^{i+1} f\,ds = a\int_{-\infty}^{+\infty} s^i f\,ds + \int_{-\infty}^{+\infty} s^i (b_0 + b_1 s + b_2 s^2)\frac{df}{ds}\,ds$$

$$\Rightarrow \quad b_0 \int_{-\infty}^{+\infty} s^i \frac{df}{ds}\,ds = -i b_0 m_{i-1}$$

$$b_1 \int_{-\infty}^{+\infty} s^{i+1} \frac{df}{ds}\,ds = -(i+1) b_1 m_i$$

$-\infty \qquad -\infty \qquad -\infty \qquad ds$

$$\int_{-\infty}^{+\infty} s^i \frac{df}{ds}\, ds = \underbrace{s^i f \Big|_{-\infty}^{+\infty}}_{0\ (f \text{ ОПАДА БРЖЕ ОД } \frac{1}{x^5} \text{ У } \pm\infty)} - i \int_{-\infty}^{+\infty} s^{i-1} f\, ds = -i\, m_{i-1}$$

$$b_2 \int_{-\infty}^{+\infty} s^{i+1} \frac{df}{ds}\, ds = -(i+2)\, b_2\, m_{i+1}$$

$$m_{i+1} = a m_i - i b_0 m_{i-1} - (i+1) b_1 m_i - (i+2) b_2 m_{i+1} \qquad i = 0, 1, 2, 3$$

$i=0 \quad \cancelto{0}{m_1} = a \cancelto{1}{m_0} - b_1 \cancelto{1}{m_0} - 2b_2 \cancelto{0}{m_1} \qquad 0 = a - b_1$

$i=1 \quad m_2 = a \cancelto{0}{m_1} - b_0 \cancelto{1}{m_0} - 2b_1 \cancelto{0}{m_1} - 3b_2 m_2$

$i=2 \quad m_3 = a m_2 - 2b_0 \cancelto{0}{m_1} - 3b_1 m_2 - 4b_2 m_3$

$i=3 \quad m_4 = a m_3 - 3b_0 m_2 - 4b_1 m_3 - 5b_2 m_4$

$$\begin{cases} m_2 = -b_0 - 3m_2 b_2 \\ m_3 = -2m_2 b_1 - 4m_3 b_2 \\ m_4 = -3m_2 b_0 - 3m_3 b_1 - 5m_4 b_2 \end{cases}$$

$m_3 = \gamma \Delta R_p^3 \qquad m_4 = \beta \Delta R_p^4$

$$\Delta = \begin{vmatrix} -1 & 0 & -3m_2 \\ 0 & -2m_2 & -4m_3 \\ -3m_2 & -3m_3 & -5m_4 \end{vmatrix} = -(10 m_2 m_4 - 12 m_3^2) + 18 m_2^3 = \Delta R_p^6 (12\gamma^2 - 10\beta + 18)$$

$$\Delta_0 = \begin{vmatrix} m_2 & 0 & -3m_2 \\ m_3 & -2m_2 & -4m_3 \\ m_4 & -3m_3 & -5m_4 \end{vmatrix} = \Delta R_p^8 (4\beta - 3\gamma^2) = \Delta R_p^6 \sigma^2 (4\beta - 3\gamma^2)$$

$$\Delta_1 = \begin{vmatrix} -1 & m_2 & -3m_2 \\ 0 & m_3 & -4m_3 \end{vmatrix} = \Delta R_p^7 \gamma (\beta + 3) = \Delta R_p^6 \sigma \gamma (\beta + 3)$$

$$\begin{vmatrix} -3m_2 & m_4 & -5m_4 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} -1 & 0 & m_2 \\ 0 & -2m_2 & m_3 \\ -3m_2 & -3m_3 & m_4 \end{vmatrix} = \Delta R_p^6 \left(2\beta - 3\gamma^2 - 6\right)$$

$$b_0 = \frac{\Delta_0}{\Delta} \qquad b_1 = \frac{\Delta_1}{\Delta} \qquad b_2 = \frac{\Delta_2}{\Delta}$$

2.7

Wednesday, March 16, 2022 10:29 AM

$$\frac{df}{ds} = \frac{s-a}{b_0 + b_2 s^2} f(s) \qquad \gamma = 0 \qquad \beta = 3 \qquad \gamma = \frac{m_3}{\Delta R_p^3} \Rightarrow m_3 = 0$$

$$sf = af + (b_0 + b_2 s^2)\frac{df}{ds} \qquad \Big/ \int_{-\infty}^{+\infty} s^i (\quad) ds$$

$$\int_{-\infty}^{+\infty} s^{i+1} f\, ds = a\int_{-\infty}^{+\infty} s^i f\, ds + \int_{-\infty}^{+\infty} s^i (b_0 + b_2 s^2)\frac{df}{ds} ds \qquad \int_{-\infty}^{+\infty} s^i \frac{df}{ds} ds = -i\, m_{i-1}$$

$$m_{i+1} = a m_i - i b_0 m_{i-1} - (i+2) b_2 m_{i+1}$$

$$i = 0: \quad \cancel{m_1}^{\,0} = a \cancel{m_0}^{\,1} - 2b_2 \cancel{m_1}^{\,0} \qquad \boxed{a = 0}$$

$$i = 1: \quad m_2 = a \cancel{m_1}^{\,0} - b_0 \cancel{m_0}^{\,1} - 3b_2 m_2 \qquad m_2 = -b_0 - 3m_2 b_2 \quad (1)$$

$$i = 2: \quad \cancel{m_3}^{\,0} = \cancel{a}^{\,0} m_2 - 2b_0 \cancel{m_1}^{\,0} - 4b_2 \cancel{m_3}^{\,0}$$

$$i = 3: \quad m_4 = a \cancel{m_3}^{\,0} - 3b_0 m_2 - 5b_2 m_4 \qquad m_4 = -3m_2 b_0 - 5m_4 b_2 \quad (2)$$

$$\beta = \frac{m_4}{\Delta R_p^4} = 3 \qquad m_4 = 3\Delta R_p^4 = 3(\Delta R_p^2)^2 = 3m_2^2$$

$$\left.\begin{aligned} m_2 &= -b_0 - 3m_2 b_2 \quad (1) \\ 3m_2^2 &= -3m_2 b_0 - 15 m_2^2 b_2 \quad (2) \end{aligned}\right] \begin{matrix} (-3m_2) \\ + \end{matrix} \Rightarrow 0 = -6m_2^2 b_2 \Rightarrow \boxed{b_2 = 0}$$

$$(1) \Rightarrow \boxed{b_0 = -m_2 = -\Delta R_p^2}$$

$$\frac{df}{ds} = \frac{s}{-\Delta R_p^2} f$$

$$\frac{df}{f} = -\frac{1}{\Delta R_p^2} s\,ds$$

$$\ln f = -\frac{1}{\Delta R_p^2}\frac{s^2}{2} + \ln C$$

$$\ln\frac{f}{C} = -\frac{1}{\Delta R_p^2}\frac{s^2}{2}$$

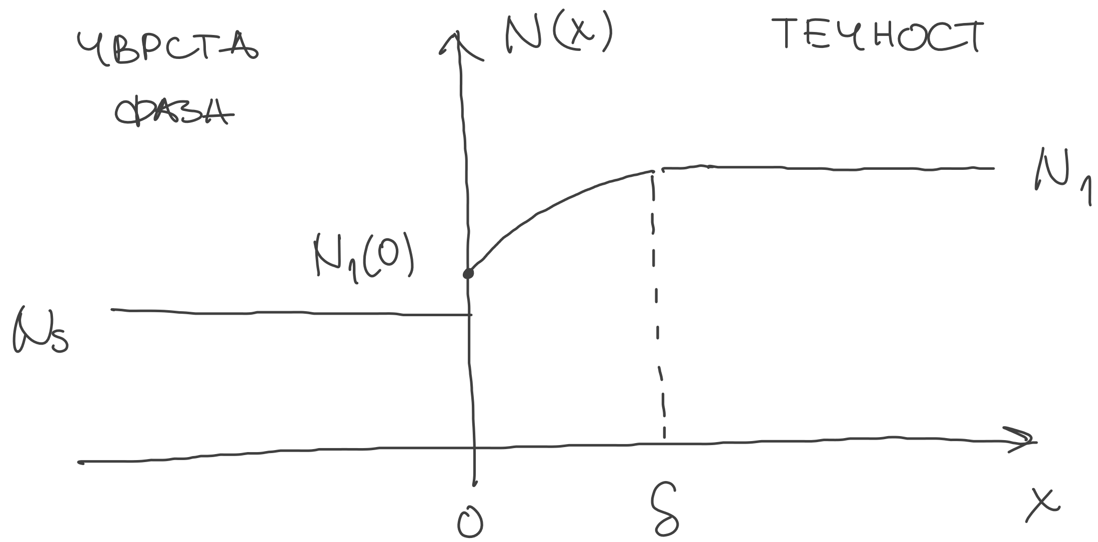
$$f(s) = Ce^{-\frac{s^2}{2\Delta R_p^2}}$$

3.1

Wednesday, April 20, 2022 9:50 AM

РАСТ СА РАВНОТЕЖНИМ ХЛАЂЕЊЕМ $\left.\frac{\partial N}{\partial t}\right|_{x=0} = const = -\alpha$

РАСТ СА НАГЛИМ ХЛАЂЕЊЕМ $N(0,t) = const$



$$\frac{\partial N}{\partial t} = D\frac{\partial^2 N}{\partial x^2}$$

$$Q = \int_0^t -\Phi\Big|_{x=0} dt = \int_0^t D\frac{\partial N}{\partial x}\Big|_{x=0} dt$$

$$x_e = \frac{Q}{N_S} \rightarrow v_e = \frac{dx_e}{dt}$$

$$\left.\frac{\partial N}{\partial t}\right|_{x=0} = -\alpha$$

$$N(0,t) = -\alpha t + const$$

$$N(x,0) = N_1 \;,\; x > 0$$

$$N(0,t) = N_1 - \alpha t$$

$$\delta N = N(x) - N_1$$

$$\frac{\partial(\delta N)}{\partial t} = D\frac{\partial^2(\delta N)}{\partial x^2}$$

$$\delta N(0,t) = -\alpha t$$

$$\delta N(x,0) = 0$$

$$\mathcal{L}\left\{\frac{\partial(\delta N)}{\partial t} = D\frac{\partial^2(\delta N)}{\partial x^2}\right\} \longrightarrow \frac{d^2 L(x,s)}{dx^2} = \frac{s}{D}L(x,s)$$

$$L(x,s) = C_1 e^{-\sqrt{\frac{s}{D}}x} + \cancel{C_2}_{0} e^{\sqrt{\frac{s}{D}}x} \qquad C_1 = L(0,s) = \mathcal{L}\{\delta N(0,t)\} = \mathcal{L}\{-\alpha t\} = -\frac{\alpha}{s^2}$$

$$L(x,s) = -\frac{\alpha}{s^2} e^{-\sqrt{\frac{s}{D}}x} \qquad \delta N(x,t) = \mathcal{L}^{-1}\{L(x,s)\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2} e^{-\sqrt{\frac{s}{a}}}\right\} = \left(\frac{1}{2a}+t\right)\operatorname{erfc}\left(\frac{1}{2\sqrt{at}}\right) - \frac{t e^{-\frac{1}{4at}}}{\sqrt{\pi}\sqrt{at}}$$

$$\frac{1}{s^2} e^{-\sqrt{\frac{s}{D}}x} = \frac{1}{s^2} e^{-\sqrt{\frac{s}{\frac{D}{x^2}}}} \Rightarrow a = \frac{D}{x^2}$$

$$\left(\frac{1}{\frac{2D}{x^2}}+t\right)\operatorname{erfc}\left(\frac{1}{\frac{2\sqrt{Dt}}{\sqrt{x^2}}}\right) - \frac{t e^{-\frac{1}{4\frac{Dt}{x^2}}}}{\sqrt{\pi}\sqrt{\frac{D}{x^2}}\sqrt{t}} = \left(\frac{x^2}{2D}+t\right)\operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) - xt\frac{e^{-\frac{x^2}{4Dt}}}{\sqrt{\pi D t}}$$

$$= \frac{1}{2D}\left[(x^2+2Dt)\operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) - 2x\frac{Dt}{\sqrt{\pi Dt}} e^{-\frac{x^2}{4Dt}}\right]$$

$$\delta N(x,t) = -\frac{\alpha}{2D}\left[(x^2+2Dt)\operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) - 2x\sqrt{\frac{Dt}{\pi}} e^{-\frac{x^2}{4Dt}}\right]$$

$$N(x,t) = N_1 - \delta N(x,t)$$

$$\frac{\partial N}{\partial x} = -\frac{\alpha}{2D}\left[2x\operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) + (x^2+2Dt)\left(-\frac{2}{\sqrt{\pi}} e^{-\frac{x^2}{4Dt}}\right)\frac{1}{2\sqrt{Dt}}\right.$$

$$-2\sqrt{\frac{Dt}{\pi}}\, e^{-\frac{x^2}{4Dt}} - 2x\sqrt{\frac{Dt}{\pi}}\, e^{-\frac{x^2}{4Dt}}\left(-\frac{2x}{4Dt}\right)\Big]$$

$$\left.\frac{\partial N}{\partial x}\right|_{x=0} = -\frac{\alpha}{\cancel{2D}}\left[\cancel{2D}t\left(-\frac{\cancel{2}}{\sqrt{\pi}}\right)\frac{1}{\cancel{2}\sqrt{Dt}} - 2\sqrt{\frac{\cancel{D}t}{\pi}}_{\sqrt{D}}\right] = \frac{\alpha t}{\sqrt{\pi D t}} + \frac{\alpha t}{\sqrt{\pi D t}} = \frac{2\alpha t}{\sqrt{\pi D t}}$$

$$Q = \int_0^t D\left.\frac{\partial N}{\partial x}\right|_{x=0} dt = \int_0^t D\,\frac{2\alpha t}{\sqrt{\pi D t}}\,dt = 2\alpha\sqrt{\frac{D}{\pi}}\int_0^t \sqrt{t}\,dt = \frac{4}{3}\alpha\sqrt{\frac{D}{\pi}}\,t^{\frac{3}{2}}$$

$$x_e = \frac{Q}{N_s} = \frac{4}{3}\frac{\alpha}{N_s}\sqrt{\frac{D}{\pi}}\,t^{\frac{3}{2}} \qquad v_e = \frac{dx_e}{dt} = \underbrace{\frac{4}{3}\frac{3}{2}}_{2}\frac{\alpha}{N_s}\sqrt{\frac{D}{\pi}}\,t^{\frac{1}{2}} \sim \sqrt{t}$$

3.2

Wednesday, April 20, 2022 10:36 AM

$N_1,\ N_1(0),\ D$

$N(0,t) = N_1(0) = \text{const}$

$\delta N(x,t) = N(x,t) - N_1$

$$\frac{\partial(\delta N)}{\partial t} = D\frac{\partial^2(\delta N)}{\partial x^2}$$

$\delta N(0,t) = N_1(0) - N_1 = \delta N_{10}$

$$\frac{d^2L}{dx^2} - \frac{s}{D}L = 0$$

$$L(x,s) = C_1 e^{-\sqrt{\frac{s}{D}}x}$$

$$L(0,s) = C_1 = \mathcal{L}\{\delta N(0,t)\} = \frac{\delta N_{10}}{s}$$

$$L(x,s) = \frac{\delta N_{10}}{s} e^{-\sqrt{\frac{s}{D}}x}$$

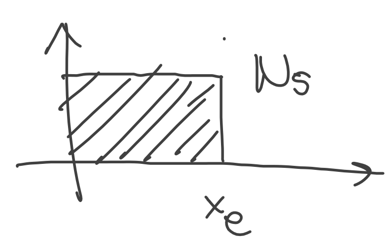
$$\mathcal{L}^{-1}\{L(x,s)\} = \delta N_{10}\,\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) = \delta N(x,t)$$

$$N(x,t) = N_1 + \delta N(x,t) = N_1 + (N_1(0) - N_1)\,\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$\frac{\partial N}{\partial x} = (N_1(0) - N_1)\left(-\frac{2}{\sqrt{\pi}}e^{-\frac{x^2}{4Dt}}\right)\frac{1}{2\sqrt{Dt}} \qquad \left.\frac{\partial N}{\partial x}\right|_{x=0} = \frac{N_1 - N_1(0)}{\sqrt{\pi D t}}$$

$$Q = \int_0^t D\left.\frac{\partial N}{\partial t}\right|_{x=0} dt = \frac{\sqrt{D}}{\sqrt{\pi}}(N_1 - N_1(0))\int_0^t \frac{dt}{\sqrt{t}} = 2\sqrt{\frac{Dt}{\pi}}(N_1 - N_1(0))$$

$$x_e = \frac{Q}{N_S} = 2\,\frac{N_1 - N_1(0)}{N_S}\sqrt{\frac{D}{\pi}}\,t^{\frac{1}{2}}$$



$Q = N_S x_e$

$$v_e = \frac{dx_e}{dt} = \frac{N_1 - N_1(0)}{\ldots}\sqrt{\frac{D}{\ldots}} \ldots \frac{1}{\ldots}$$

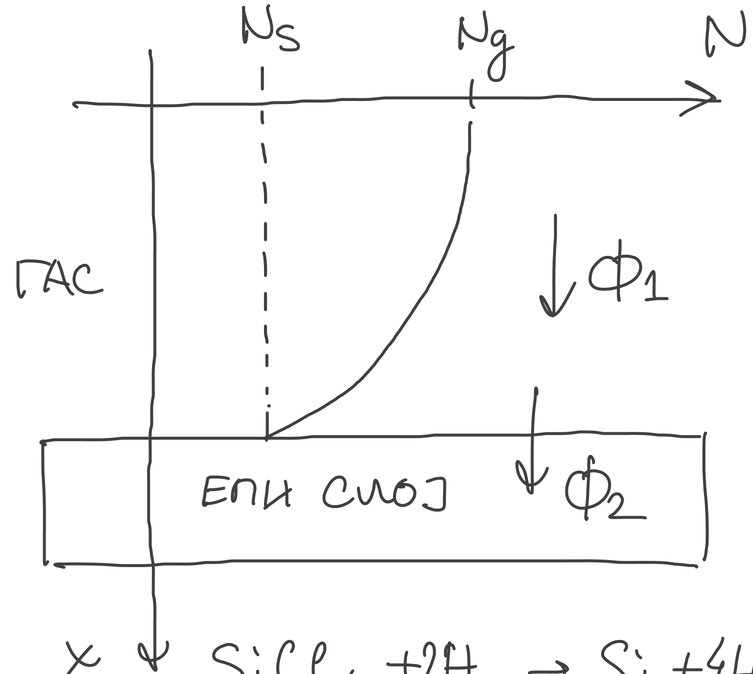
$dt \quad N_s \quad \sqrt{\pi t} \quad \sqrt{t}$

3.3

Tuesday, March 22, 2022 5:52 PM

k_s, h_g, N_g, N_a

$k_s \ll h_g, \quad h_g \ll k_s$



$SiCl_4 + 2H_2 \rightarrow Si + 4HCl$

$\Phi_1 = h_g(N_g - N_s)$

$\Phi_2 = k_s N_s$

$\Phi_1 = \Phi_2 = \Phi$

$h_g(N_g - N_s) = k_s N_s$

$(h_g + k_s) N_s = h_g N_g$

$N_s = \dfrac{h_g}{h_g + k_s} N_g$

$\Phi = k_s N_s = \dfrac{k_s h_g}{h_g + k_s} N_g$

$\Phi = N_a v$

$v = \dfrac{k_s h_g}{k_s + h_g} \dfrac{N_g}{N_a}$

① $k_s \ll h_g$

$v \approx k_s \dfrac{N_g}{N_a}$

НИСКЕ Т

$- \dfrac{E_a}{\ldots}$

② $h_g \ll k_s$

$v \approx h_g \dfrac{N_g}{N_a}$

ВИСОКЕ Т

$k_g \sim e \quad k_B T$

$h_g \neq h_g(T)$

$T \downarrow \quad \frac{E_a}{k_B T} \uparrow \quad e^{-\frac{E_a}{k_B T}} \downarrow$

3.4

Wednesday, March 23, 2022 9:43 AM

(a) $v = ?$ $SiCl_4$

$T = 1200°C$

$h_g = 5\ cm/s$

$k_{s0} = 10^7\ cm/s$

$E_a = 1{,}9\ eV$

$N_g = 3 \cdot 10^{16}\ cm^{-3}$

$N_a = 5 \cdot 10^{22}\ cm^{-3}$

(a) $$v = \frac{k_s h_g}{k_s + h_g} \frac{N_g}{N_a}$$

$$k_s = k_{s0} e^{-\frac{E_a}{k_B T}} = 3{,}2 \frac{cm}{s}$$

$$v = 1{,}17 \cdot 10^{-6} \frac{cm}{s} = 0{,}703 \frac{\mu m}{min}$$

(b) $\frac{\Delta T}{T} = 1\%$ $\frac{\Delta v}{v} = ?$

$$\Delta k_s = \underbrace{k_{s0} e^{-\frac{E_a}{k_B T}}} \frac{E_a}{k_B T^2} \Delta T = k_s \frac{E_a}{k_B T^2} \Delta T$$

$$\Delta v = \frac{N_g}{N_a} h_g \frac{k_s + h_g - k_s}{(k_s + h_g)^2} \Delta k_s$$

$$\frac{\Delta v}{v} = \frac{\cancel{\frac{N_g}{N_a}} h_g \frac{h_g}{(k_s + h_g)^{\cancel{2}}} \Delta k_s}{\cancel{\frac{N_g}{N_a}} h_g \frac{k_s}{\cancel{k_s + h_g}}} = \frac{h_g}{k_s (k_s + h_g)} \Delta k_s$$

$$\frac{\Delta v}{v} = \frac{h_g}{\cancel{k_s}(k_s + h_g)} \cancel{k_s} \frac{E_a}{k_B T^2} \Delta T$$ $\frac{\Delta T}{T} = 1\%$

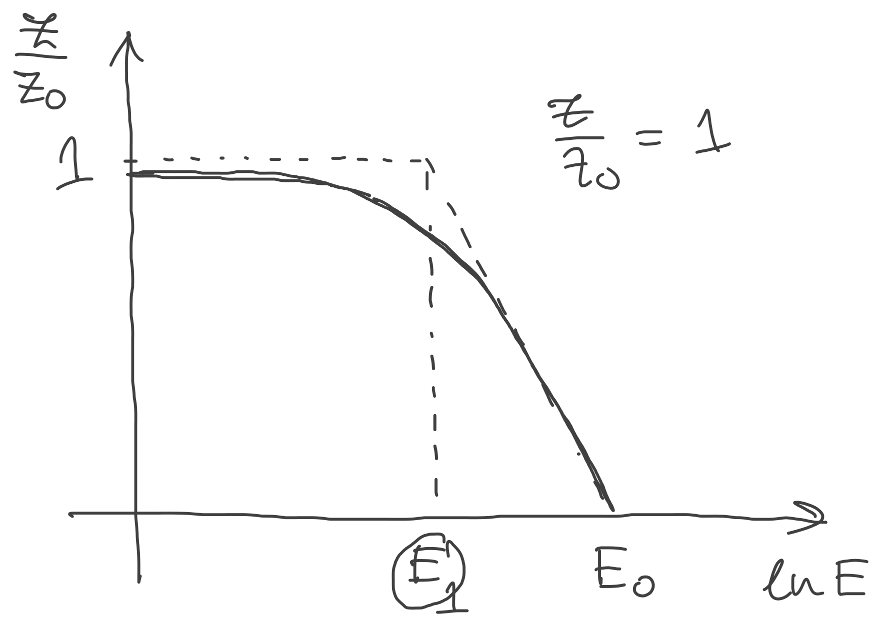
$$\frac{\Delta v}{v} = \frac{h_g}{k_s + h_g} \frac{E_a}{k_B T} \frac{\Delta T}{T} = 9{,}11\%$$

4.2

Wednesday, March 23, 2022 10:52 AM

$E_0 = 100\ mJ/cm^2 \quad t = 1s$

$\gamma = 4 \quad I_{min} = ?$



E_0 – ЕНЕРГИЈА ПРАГА

$$\gamma = \frac{1}{\ln \frac{E_0}{E_1}}$$ КОНТРАСТ

$$I_0 = \frac{E_0}{t} = 100\ \frac{mW}{cm^2}$$

$$\ln \frac{E_0}{E_1} = \frac{1}{\gamma}$$

$$E_0 = E_1 e^{\frac{1}{\gamma}}$$

$$E_1 = E_0 e^{-\frac{1}{\gamma}} \quad / : t$$

$$I_1 = \frac{E_0}{t} e^{-\frac{1}{\gamma}} = I_0 e^{-\frac{1}{\gamma}} = 77{,}9\ \frac{mW}{cm^2}$$

4.5

Wednesday, March 23, 2022 10:43 AM

$NA = 0,4$

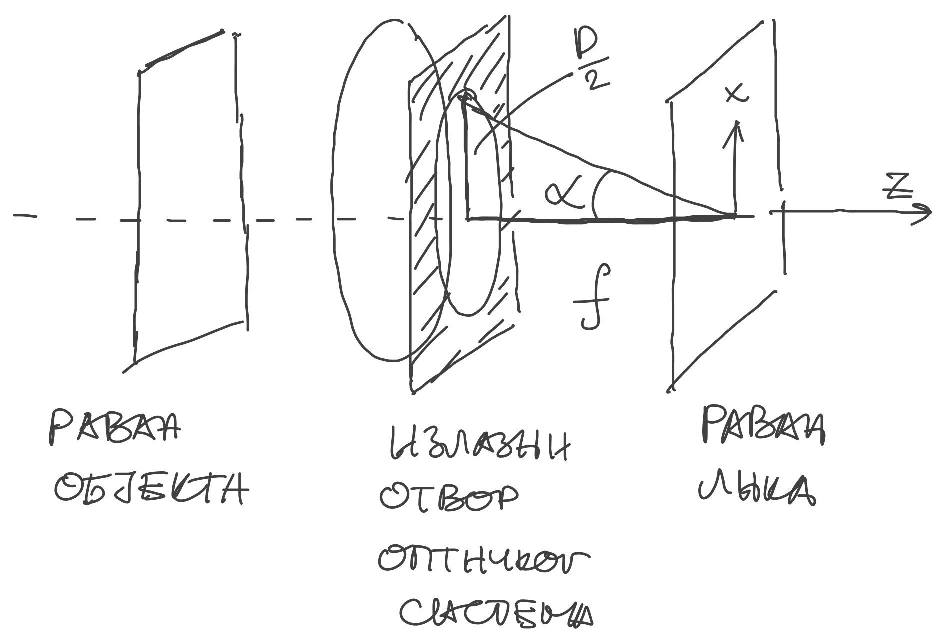
$D = 0,5\,cm$

$\lambda = 600\,nm$

$\Delta x = ?$

$\Delta Z = ?$

ОПТИЧКИ ПРОЈЕКЦИОНИ СИСТЕМ



$$\Delta x = \frac{0,61\lambda}{NA} = \frac{0,61 \cdot 600}{0,4} = 0,915\,\mu m$$

$$NA = n \sin\alpha \approx n\frac{D}{2f} \qquad n = 1$$

$$NA = \frac{D}{2f}$$

$$f = \frac{D}{2NA} = \frac{0,5}{0,8} = 0,625\,cm$$

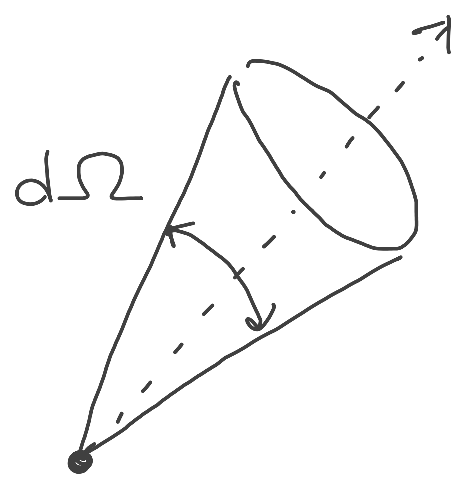
$$\Delta Z = \frac{n\lambda}{2(NA)^2} = \frac{600}{2 \cdot 0,4^2} = 1,875\,\mu m$$

5.2

Wednesday, March 23, 2022 9:56 AM

$\dot{m}$ МАСЕНИ ТОК

ТАЧКАСТИ ИЗВОР



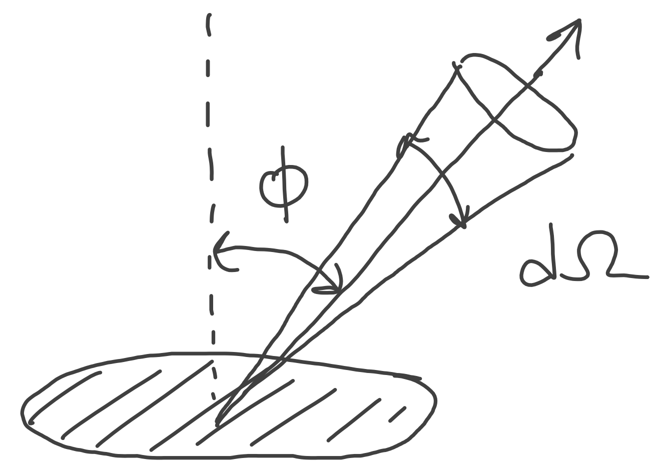
$$\frac{d\dot{m}}{\dot{m}} = \frac{d\Omega}{4\pi}$$

(ИЗОТРОПАН)

$d\dot{m} = ?$ $\dot{m} \rightarrow 2\pi$

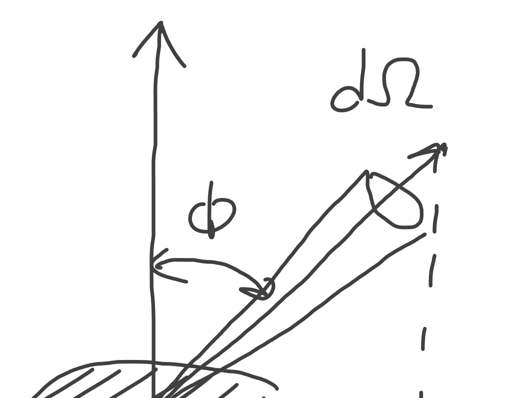
$d\dot{m} = \dot{m}_{max} \cos\phi \, d\Omega$

ПОВРШИНСКИ ИЗВОР

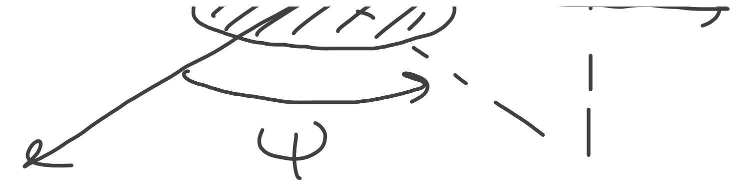


$d\dot{m} = \dot{m}_{max} \cos\phi \, d\Omega$

ЛАМБЕРОВ ЗАКОН



$$\dot{m} = \dot{m}_{max} \int_{\Omega} \cos\phi \, d\Omega$$



$$d\Omega = \sin\phi \, d\phi \, d\psi$$

$$\dot{m} = \dot{m}_{max} \underbrace{\int_{\psi=0}^{2\pi} d\psi}_{2\pi} \; \underbrace{\int_{\phi=0}^{\frac{\pi}{2}} \sin\phi \cos\phi \, d\phi}$$

$$\int_{\phi=0}^{\frac{\pi}{2}} \sin\phi \, d(\sin\phi) = \frac{\sin^2\phi}{2}\Bigg|_{0}^{\frac{\pi}{2}} = \frac{1}{2}$$

$$\dot{m} = \dot{m}_{max} \, 2\pi \, \frac{1}{2} \Rightarrow \dot{m}_{max} = \frac{\dot{m}}{\pi}$$

$$\boxed{d\dot{m} = \frac{\dot{m}}{\pi} \cos\phi \, d\Omega}$$

5.3

Wednesday, March 23, 2022 10:22 AM

$\dot{\delta}$ dA

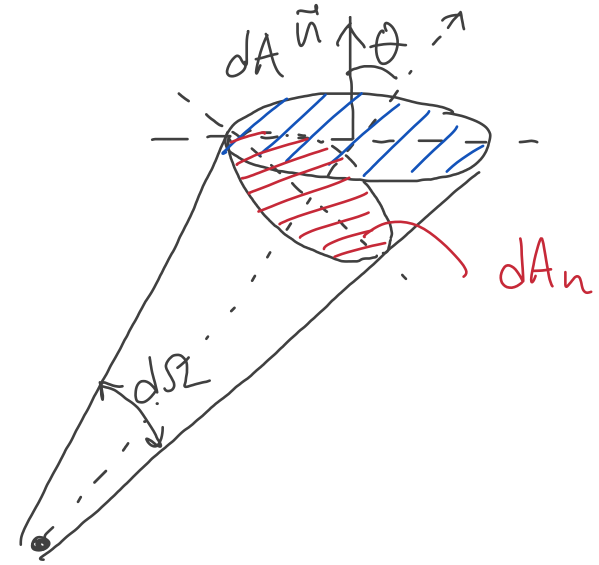
(а) ТАЧКАСТИ И.

(б) ПОВРШИНСКОГ И.

$\rho, \dot{m}, r, \theta$

$$dm = \rho dV = \rho \dot{\delta} dA$$

(а)



$$\frac{dm}{m} = \frac{d\Omega}{4\pi}$$

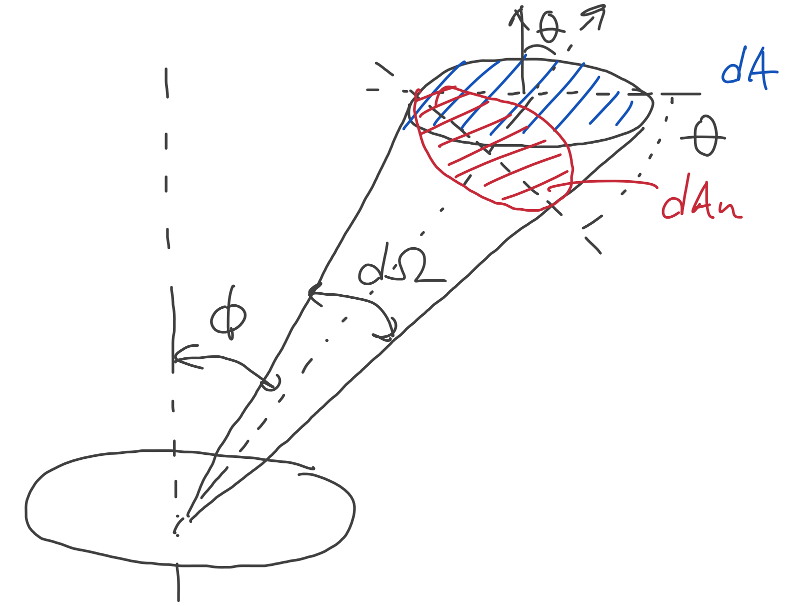
$$dm = \frac{d\Omega}{4\pi} m$$

$$d\Omega = \frac{dA_n}{r^2} = \frac{dA \cos\theta}{r^2}$$

$$dm = \frac{\dot{m}}{4\pi} \frac{\cos\theta}{r^2} dA$$

$$\dot{\delta} = \frac{dm}{\rho dA} = \frac{\dot{m}}{4\pi\rho} \frac{\cos\theta}{r^2}$$

(б)



5.2. → $dm = \frac{\dot{m}}{\pi} \cos\phi \, d\Omega$

$$dm = \frac{\dot{m}}{\pi} \cos\phi \frac{dA \cos\theta}{r^2}$$

$$\dot{\delta} = \frac{dm}{\rho dA} = \frac{\dot{m}}{\pi\rho} \frac{\cos\phi \cos\theta}{r^2}$$

7.11

Wednesday, March 30, 2022 1:02 AM

$$N_B(x) = N_B(0)\, e^{-a\frac{x}{x_B}}$$

$$\delta n_B(x) = ? \qquad J_{Bn}(x)$$

$$V_{BE} \gg V_t, \quad |V_{BC}| \gg V_t$$

$$L_B \to \infty$$

$$J_{Bn}(x) = -q\mu_B n_B \mathcal{E} - qD_B \frac{dn_B}{dx} \qquad \frac{D}{\mu} = V_t$$

РЕФЕРЕНТНИ СМЕР СУПРОТАН СМЕРУ x-ОСЕ

$$J_{Bn}(x) = -qD_B\left(\frac{\mathcal{E}}{V_t} n_B + \frac{dn_B}{dx}\right)$$

$$n_B(x) = n_{B0}(x) + \delta n_B(x)$$

$$J_{Bn}(x) = -qD_B\left(\underbrace{\frac{\mathcal{E}}{V_t} n_{B0}(x) + \frac{dn_{B0}}{dx}}_{=0} + \frac{\mathcal{E}}{V_t}\delta n_B(x) + \frac{d\delta n_B(x)}{dx}\right)$$

$$\mathcal{E} = \underbrace{\frac{V_t}{N_B}\frac{dN_B}{dx}} = -V_t\frac{a}{x_B} \qquad a = \ln\left(\frac{N_B(0)}{N_B(x_B)}\right) = \ln K$$

$$\frac{\mathcal{E}}{V_t} n_{B0} + \frac{dn_{B0}}{dx} = \frac{1}{N_B}\frac{dN_B}{dx} n_{B0} + \frac{dn_{B0}}{dx} = \frac{1}{N_B}\left[n_{B0}\frac{dN_B}{dx} + N_B\frac{dn_{B0}}{dx}\right]$$

$$= \frac{1}{N_B}\frac{d}{dx}(\underbrace{n_{B0}N_B}_{n_i^2}) = 0 \qquad n_{B0}N_B = n_{B0}p_{B0} = n_i^2$$

$$-\frac{J_{Bn}}{qD_B} = \frac{d\delta n_B}{dx} + \frac{\mathcal{E}}{V_t}\delta n_B \;\Big/ \frac{d}{dx} \qquad \frac{1}{q}\frac{dJ_{Bn}}{dx} + \frac{n_B(x) - n_{B0}(x)}{\tau_B} = 0$$

$$\frac{1}{q}\frac{dJ_{Bn}}{dx} = -\frac{\delta n_B}{\tau_B}$$

$$-\frac{1}{q}\frac{1}{D_B}\frac{dJ_{Bn}}{dx} = \frac{d^2\delta n_B}{dx^2} + \frac{\mathcal{E}}{V_t}\frac{d\delta n_B}{dx}$$

$$\frac{\delta n_B}{D_B\tau_B} = \frac{d^2\delta n_B}{dx^2} + \frac{\mathcal{E}}{V_t}\frac{d\delta n_B}{dx}$$

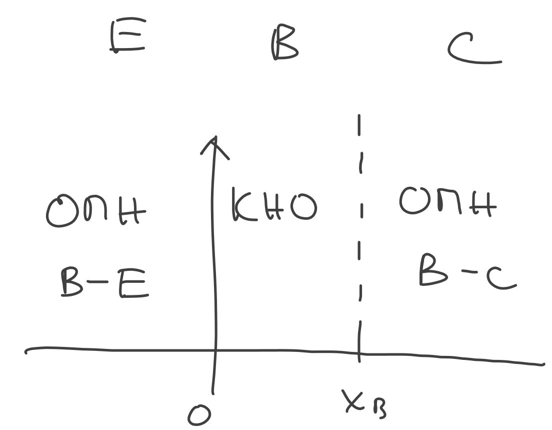
$$\frac{d^2\delta n_B}{dx^2} + \frac{\mathcal{E}}{V_t}\frac{d\delta n_B}{dx} - \frac{\delta n_B}{L_B^2} = 0$$

$$\delta n_B(x) = C_1 e^{s_+ x} + C_2 e^{s_- x} \qquad s_\pm = -\frac{\mathcal{E}}{2V_t} \pm \sqrt{\left(\frac{\mathcal{E}}{2V_t}\right)^2 + \frac{1}{L_B^2}}$$

$$s_\pm = -\nu \pm \lambda$$

$$\nu = \frac{\mathcal{E}}{2V_t} \qquad \lambda = \sqrt{\nu^2 + L_B^{-2}}$$

$$\nu = -V_t \frac{a}{x_B}\frac{1}{2V_t} = -\frac{a}{2x_B} = -\frac{\ln K}{2x_B}$$



ЗАКОН СПОЈА $n_p(x_p) = n_{p0} e^{\frac{V}{V_t}}$

$$\delta n_p(x_p) = n_{p0}\left(e^{\frac{V}{V_t}} - 1\right)$$

$$\delta n_B(0) = \frac{n_i^2}{N_B(0)}\left(e^{\frac{V_{BE}}{V_t}} - 1\right) = C_1 + C_2 \quad (1)$$

$$\delta n_B(x_B) = \frac{n_i^2}{N_B(x_B)}\left(e^{\frac{V_{BC}}{V_t}} - 1\right) = C_1 e^{s_+ x_B} + C_2 e^{s_- x_B} \quad (2)$$

$$C_2 = \delta n_B(0) - C_1$$

$$\delta n_B(x_B) = C_1 e^{s_+ x_B} - C_1 e^{s_- x_B} + \delta n_B(0) e^{s_- x_B}$$

$$\delta n_B(x_B) - \delta n_B(0) e^{-\nu x_B} e^{-\lambda x_B} = C_1 e^{-\nu x_B} \underbrace{\left[e^{\lambda x_B} - e^{-\lambda x_B}\right]}_{2\,\mathrm{sh}(\lambda x_B)}$$

$$C_1 = \delta n_B(x_B) e^{\nu x_B} - \delta n_B(0) e^{-\lambda x_B}$$

$$\frac{\quad}{2\operatorname{sh}(\lambda x_B)}$$

$$C_2 = \frac{\delta n_B(0)e^{\lambda x_B} - \cancel{\delta n_B(0)e^{-\lambda x_B}} - \delta n_B(x_B)e^{\nu x_B} - \cancel{\delta n_B(0)e^{-\lambda x_B}}}{2\operatorname{sh}(\lambda x_B)}$$

$$\delta n_B(x) = C_1 e^{-\nu x}\, e^{\lambda x} + C_2 e^{-\nu x}\, e^{-\lambda x}$$

$$\delta n_B(x) = \frac{\delta n_B(0)}{2\operatorname{sh}(\lambda x_B)} e^{-\nu x}\Big[\underbrace{-e^{\lambda(x_B-x)} + e^{\lambda(x_B-x)}}_{2\operatorname{sh}[\lambda(x_B-x)]}\Big] + \frac{\delta n_B(x_B)}{2\operatorname{sh}(\lambda x_B)} e^{\nu(x_B-x)}\Big[\underbrace{e^{\lambda x} - e^{-\lambda x}}_{2\operatorname{sh}(\lambda x)}\Big]$$

$$\delta n_B(x) = \delta n_B(0)\, e^{-\nu x}\, \frac{\operatorname{sh}[\lambda(x_B-x)]}{\operatorname{sh}(\lambda x_B)} + \delta n_B(x_B)\, e^{\nu(x_B-x)}\, \frac{\operatorname{sh}(\lambda x)}{\operatorname{sh}(\lambda x_B)}$$

$$J_{Bn}(x) = -qD_B\left(2\nu\, \delta n_B + \frac{d\delta n_B}{dx}\right) \qquad \nu = \frac{\mathcal{E}}{2V_t}$$

$$\frac{d\delta n_B}{dx} = \delta n_B(0) e^{-\nu x} (-\nu) \frac{\operatorname{sh}[\lambda(x_B-x)]}{\operatorname{sh}(\lambda x_B)} + \delta n_B(0) e^{-\nu x} (-\lambda) \frac{\operatorname{ch}[\lambda(x_B-x)]}{\operatorname{sh}(\lambda x_B)}$$

$$+ \delta n_B(x_B) e^{\nu(x_B-x)} (-\nu) \frac{\operatorname{sh}(\lambda x)}{\operatorname{sh}(\lambda x_B)} + \delta n_B(x_B) e^{\nu(x_B-x)} \lambda \frac{\operatorname{ch}(\lambda x)}{\operatorname{sh}(\lambda x_B)}$$

$$J_{Bn}(x) = -\frac{qD_B}{\operatorname{sh}(\lambda x_B)}\Big\{\delta n_B(0)\, e^{-\nu x}\Big[\nu \operatorname{sh}(\lambda(x_B-x)) - \lambda \operatorname{ch}(\lambda(x_B-x))\Big]$$

$$+ \delta n_B(x_B) e^{\nu(x_B - x)} \left[\nu \, sh(\lambda x) + \lambda \, ch(\lambda x) \right] \Big\}$$

$L_B \to \infty$, $\lambda \to \nu$ $\left(\lambda = \sqrt{\nu^2 + L_B^{-2}} \right)$

ΔAP $V_{BE} >> V_t$, $|V_{BC}| >> V_t$ $\quad e^{\frac{V_{BE}}{V_t}} - 1 \approx e^{\frac{V_{BE}}{V_t}}$, $e^{\frac{V_{BC}}{V_t}} - 1 \approx -1$

$$\delta n_B(x) = \frac{n_i^2}{N_B(0)} e^{\frac{V_{BE}}{V_t}} \frac{1 - e^{a\left(\frac{x}{x_B} - 1\right)}}{1 - e^{-a}} \qquad J_{Bn}(x) = q D_B \delta n_B(0) \frac{a}{2x_B} \frac{e^{\frac{a}{2}}}{sh\left(\frac{a}{2}\right)} = const$$

7.12

Wednesday, March 30, 2022 1:02 AM

$J_C = ?$ $n_B(x) = ?$

при $N_B(x)$

ДАР

$$J_{Bp} = q\mu_{Bp} p_B \mathcal{E} - qD_{Bp}\frac{dp_B}{dx} = 0$$ (У СМЕРУ x-ОСЕ)

$$\mathcal{E} = \frac{D_{Bp}}{\mu_{Bp}}\frac{1}{p_{Bp}}\frac{dp_B}{dx} = V_t \frac{1}{N_B}\frac{dN_B}{dx}$$

$$J_{Bn} = q\mu_B n_B \mathcal{E} + qD_B\frac{dn_B}{dx} = qD_B\left(\frac{\mu_B}{D_B} n_B V_t \frac{1}{N_B}\frac{dN_B}{dx} + \frac{dn_B}{dx}\right)$$

$$= \frac{qD_B}{N_B}\left(n_B\frac{dN_B}{dx} + N_B\frac{dn_B}{dx}\right) = \frac{qD_B}{N_B}\frac{d(n_B N_B)}{dx}$$

$J_{Bn} = const$ АКО НЕМА РЕКОМБИНАЦИЈЕ

$$\frac{J_{Bn}}{q}\int_x^{x_B}\frac{N_B}{D_B}dx = \int_x^{x_B} d(n_B N_B)$$

$$J_{Bn} = \frac{q}{\int_x^{x_B}\frac{N_B}{D_B}dx}\int_x^{x_B} d(n_B N_B)$$

$J_{Bn} = const$

$x = 0$ БИРАМО ЗА ДОЊУ ГРАНИЦУ

$$J_{Bn} = \frac{q}{\underbrace{\int_0^{x_B}\frac{N_B}{D_B(N_B)}dx}}\left[n_B(x_B)N_B(x_B) - n_B(0)N_B(0)\right]$$

G_B БАЗНИ ГУМЕЛОВ БРОЈ

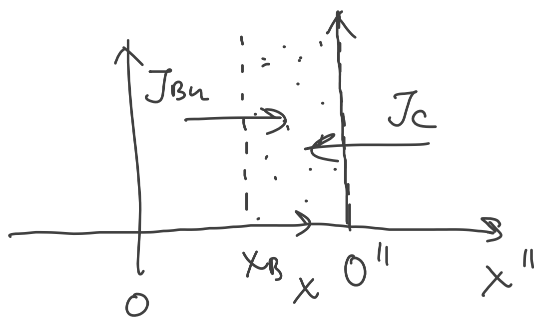
$$n_B(0)N_B(0) = \frac{n_i^2}{N_B(0)} e^{\frac{V_{BE}}{V_t}} N_B(0) = n_i^2 e^{\frac{V_{BE}}{V_t}}$$

$$n_B(x_B)N_B(x_B) = \frac{n_i^2}{N_B(x_B)} e^{\frac{V_{BC}}{V_t}} N_B(x_B) = n_i^2 e^{\frac{V_{BC}}{V_t}}$$

$$J_{Bn} = \frac{q n_i^2}{G_B}\left[e^{\frac{V_{BC}}{V_t}} - e^{\frac{V_{BE}}{V_t}}\right]$$

Moll – Ross-ова релација

$$J_C = -J_{Bn}$$



$$J_C = \frac{q n_i^2}{G_B}\left[e^{\frac{V_{BE}}{V_t}} - e^{\frac{V_{BC}}{V_t}}\right]$$

У ДАР и $V_{BE} \gg V_t$ и $|V_{BC}| \gg V_t$

$$\boxed{J_C = \frac{q n_i^2}{G_B} e^{\frac{V_{BE}}{V_t}}}$$

$$-\frac{J_C}{q}\int_x^{x_B}\frac{N_B}{D_B}dx = \int_x^{x_B} d(n_B N_B) = \underbrace{\cancel{n_B(x_B)}}_{\approx 0} N_B(x_B) - n_B(x) N_B(x)$$

$$\boxed{n_B(x) = \frac{J_C}{q N_B}\int_x^{x_B}\frac{N_B}{D_B}dx}$$

ЗА УНИФОРМНО ДОПИРАЊЕ

$$n_B(x) = \frac{J_C}{q D_B}(x_B - x) = n_B(0)\left(1 - \frac{x}{x_B}\right)$$

$$n_B(0) = \frac{J_C x_B}{q D_B}$$

ZWB

7.13

Wednesday, April 6, 2022 8:16 AM

J_c, $n_B(x)$ $(V_{BE} \gg V_t, |V_{BC}| \gg V_t)$

(a) $N_{B\,exp} = N_B(0)\, e^{-a\frac{x}{x_B}}$

(b) $N_{B\,Gauss}$

$n_B(x)/n_B(0)$ (b) $R_p = \frac{x_B}{2}$

$K = (1, 10, 100)$ $K = \frac{N_B(0)}{N_B(x_B)}$

$a = \ln K = \ln\left(\frac{N_B(0)}{N_B(x_B)}\right)$

(a) $N_B(x) = N_B(0)\, e^{-a\frac{x}{x_B}}$

7.12. $\rightarrow$ $J_C = \frac{q n_i^2}{G_B} e^{\frac{V_{BE}}{V_t}}$

$$G_B = \int_0^{x_B} \frac{N_B}{D_B} dx = \frac{N_B(0)}{D_B} \int_0^{x_B} e^{-a\frac{x}{x_B}} dx$$

$$G_B = \frac{N_B(0)}{D_B} \frac{x_B}{a} e^{-a\frac{x}{x_B}} \Big|_{x_B}^{0} = \frac{N_B(0)\, x_B}{D_B\, a}\left[1 - e^{-a}\right]$$

$$J_C = \frac{q n_i^2 D_B}{N_B(0)} \frac{a}{x_B} \frac{e^{\frac{V_{BE}}{V_t}}}{1 - e^{-a}}$$

$$n_B(x) = \frac{J_c}{q N_B(x)} \int_x^{x_B} \frac{N_B(x)}{D_B} dx$$

$$n_B(x) = \frac{1}{\cancel{q} N_B(x)} \frac{\cancel{q} n_i^2 \cancel{D_B}}{\cancel{N_B(0)}} \frac{\cancel{a}}{\cancel{x_B}} \frac{e^{V_{BE}/V_t}}{1 - e^{-a}} \frac{\cancel{N_B(0)}}{\cancel{D_B}} \underbrace{\int_x^{x_B} e^{-a\frac{x}{x_B}} dx}_{\frac{\cancel{x_B}}{\cancel{a}}\left(e^{-a\frac{x}{x_B}} - e^{-a}\right)}$$

$$n_B(x) = \frac{n_i^2}{N_B(0)} \frac{e^{a\frac{x}{x_B}}\left(e^{-a\frac{x}{x_B}} - e^{-a}\right)}{1 - e^{-a}} e^{\frac{V_{BE}}{V_t}} = \frac{n_i^2}{N_B(0)} \frac{1 - e^{a\left(\frac{x}{x_B} - 1\right)}}{1 - e^{-a}} e^{\frac{V_{BE}}{V_t}}$$

Z. 11. $n_B(x) \approx \delta n_B(x)$

(б) 2.5. $N_G = N_B(0)\, e^{-\frac{x(x+R_p)}{2\Delta R_p^2}}$

$$G_B = \int_0^{x_B} \frac{N_B}{D_B}dx = \frac{N_B(0)}{D_B}\int_0^{x_B} e^{-\frac{(x+R_p)^2}{2\Delta R_p^2}}\, e^{\frac{R_p^2}{2\Delta R_p^2}}\, dx$$

$$= \frac{N_B(0)}{D_B}\, e^{\frac{R_p^2}{2\Delta R_p^2}}\left[\int_0^{+\infty} e^{-\frac{(x+R_p)^2}{2\Delta R_p^2}}dx - \int_{x_B}^{+\infty} e^{-\frac{(x+R_p)^2}{2\Delta R_p^2}}dx\right]$$

$u = \frac{x+R_p}{\sqrt{2}\Delta R_p}$

$dx = \sqrt{2}\Delta R_p\, du$

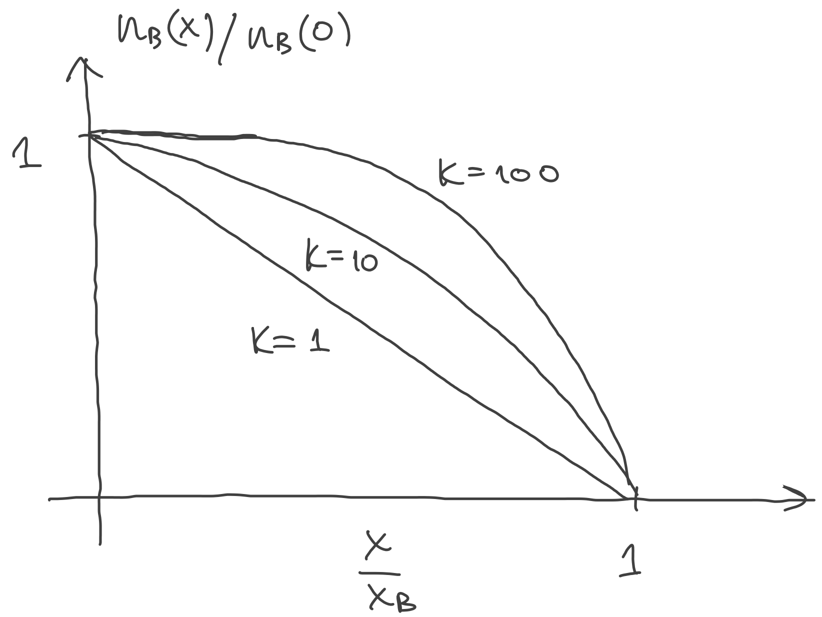
$$G_B = \boxed{\frac{2}{\sqrt{\pi}}}\frac{\sqrt{\pi}}{\cancel{2}}\frac{N_B(0)}{D_B}\, e^{\frac{R_p^2}{2\Delta R_p^2}}\cancel{\sqrt{2}}\,\Delta R_p\left[\int_{\frac{R_p}{\sqrt{2}\Delta R_p}}^{+\infty} e^{-u^2}du - \int_{\frac{x_B+R_p}{\sqrt{2}\Delta R_p}}^{+\infty} e^{-u^2}du\right]$$

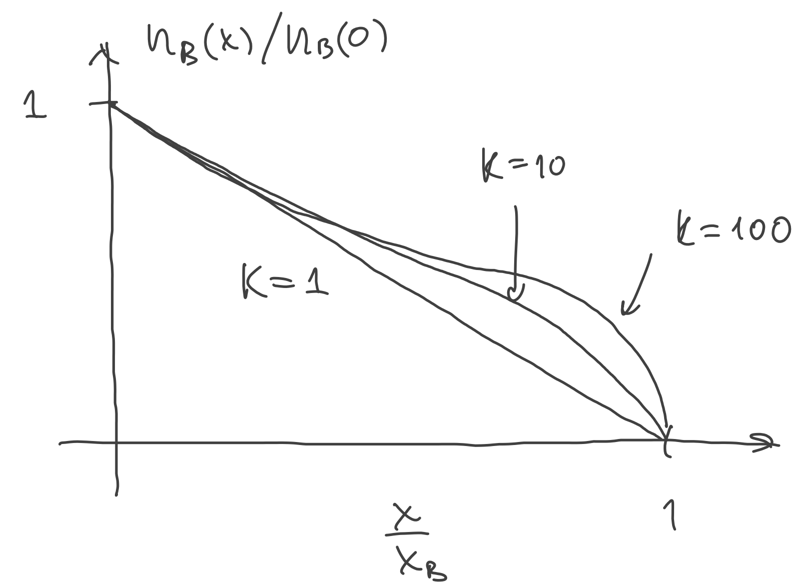
$$G_B = \sqrt{\frac{\pi}{2}}\,\frac{N_B(0)}{D_B}\, e^{\frac{R_p^2}{2\Delta R_p^2}}\,\Delta R_p\left[\operatorname{erfc}\left(\frac{R_p}{\sqrt{2}\Delta R_p}\right) - \operatorname{erfc}\left(\frac{x_B+R_p}{\sqrt{2}\Delta R_p}\right)\right]$$

$$J_C = \sqrt{\frac{2}{\pi}}\,\frac{q n_i^2 D_B}{N_B(0)\Delta R_p}\, e^{-\frac{R_p^2}{2\Delta R_p^2}}\left[\quad\right]^{-1} e^{\frac{V_{BE}}{V_t}}$$

$$n_B(x) = \frac{J_C}{q N_B(x)}\int^{x_B} \frac{N_B(x)}{D_B}dx$$

$$= \frac{J_c}{q D_B} \frac{1}{N_B(0)} e^{\frac{x(x+R_p)}{2\Delta R_p^2}} N_B(0) \underbrace{\int_x^{x_B} e^{-\frac{x(x+R_p)}{2\Delta R_p^2}} dx}_{\sqrt{\frac{\pi}{2}}\,\Delta R_p\, e^{\frac{R_p^2}{2\Delta R_p^2}} \left[\operatorname{erfc}\left(\frac{x+R_p}{\sqrt{2}\Delta R_p}\right) - \operatorname{erfc}\left(\frac{x_B+R_p}{\sqrt{2}\Delta R_p}\right)\right]}$$





$$K = \frac{N_B(0)}{N_B(x_B)} = 1 \qquad K = 1 \Rightarrow a = \ln K = 0$$

$$N_B(x) = N_B(0)\, e^{-a\frac{x}{x_B}}$$

$$a = 0 \Rightarrow N_B(x) = N_B(0)$$

7.13.

2.5. $$\Delta R_p^2 = \frac{x_B(x_B + 2R_p)}{2a} \longleftarrow R_p = \frac{x_B}{2}$$

$$\Delta R_p = \frac{x_B}{\sqrt{a}} \xrightarrow{a=0} \Delta R_p \to \infty$$

УНИФОРМНО ДОПИРАЊЕ

7.12. $$\frac{n_B(x)}{n_B(0)} = 1 - \frac{x}{x_B}$$

7.15

Wednesday, April 6, 2022 9:52 AM

$\beta_F = ?$ $\quad x_B, x_E \quad Q_B = 10^{12}\,\mathrm{cm}^{-2} \quad \frac{D_B}{D_E} = 2$

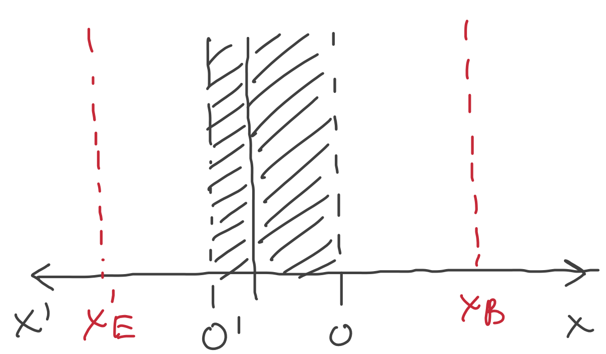
npn $\quad D_B, D_E \quad Q_E = 10^{14}\,\mathrm{cm}^{-2}$

$$\beta_F = \frac{\alpha_F}{1-\alpha_F}$$ α_F П. У СПОЈУ СА ЗАЈ. Б.

$\alpha_F = \alpha_T \gamma \qquad \alpha_T = \frac{J_{Cn}}{J_{En}}$ БАЗНИ ТР. ФАКТОР $\qquad \gamma = \frac{J_{En}}{J_{En}+J_{Ep}}$ ЕФИКАСНОСТ ЕМИТОРА

НЕМА РЕКОМБИНАЦИЈЕ $\Rightarrow J_{Cn} = J_{En} \Rightarrow \alpha_T = 1$

$\alpha_F = \gamma \qquad \beta_F = \frac{\gamma}{1-\gamma} = \frac{1}{\frac{1}{\gamma} - 1} \qquad \gamma = \frac{J_{Cn}}{J_{Cn}+J_{Ep}} = \frac{J_C}{J_C + \boxed{J_{Ep}}}$ 7.12.



$$\mathcal{E} = -\frac{V_t}{N_E}\frac{dN_E}{dx'}$$

$$\gamma = \frac{1}{1 + \frac{J_{Ep}}{J_C}} \qquad J_{Ep} = ?$$

$$J_{Ep} = q p_E(x') \mu_{Ep} \mathcal{E} - q D_{Ep} \frac{dp_E(x')}{dx'} \qquad J_{Ep} = -\frac{qD_E}{N_E}\frac{d}{dx'}\left(p_E(x') N_E(x')\right)$$

$$J_{Ep} = qD_E\left(\frac{\mathcal{E}}{V_t} p_E(x') - \frac{dp_E(x')}{dx'}\right)$$

$$J_{Ep} = q D_E \left(-\frac{1}{N_E} \frac{dN_E}{dx'} p_E - \frac{dp_E}{dx'} \right)$$

$$-\frac{J_{Ep}}{q} \int_{0'}^{x_E'} \frac{N_E}{D_E} dx' = p_E(x_E') N_E(x_E') - p_E(0') N_E(0')$$

$$p_E(0') n_E(0') = n_i^2 = p_E(0') n_{E0}(0') e^{-\frac{V_{BE}}{V_t}} = p_E(0') N_E(0') e^{-\frac{V_{BE}}{V_t}} \Rightarrow p_E(0') N_E(0') = n_i^2 e^{\frac{V_{BE}}{V_t}}$$

$$p_E(x_E') n_E(x_E') = n_i^2 = p_E(x_E') \underbrace{n_{E0}(x_E')} = \underbrace{p_E(x_E') N_E(x_E')}$$

Е.Г. НА КОНТАКТУ

ЈЕ У РАВНОТЕЖИ

$$-\frac{J_{Ep}}{q} \underbrace{\int_{0'}^{x_E'} \frac{N_E}{D_E} dx'}_{G_E} = n_i^2 - n_i^2 e^{\frac{V_{BE}}{V_t}} \Rightarrow J_{Ep} = \frac{q n_i^2}{G_E} \left(e^{\frac{V_{BE}}{V_t}} - 1 \right) \qquad V_{BE} \gg V_t$$

$$\boxed{J_{Ep} = \frac{q n_i^2}{G_E} e^{\frac{V_{BE}}{V_t}}} \qquad J_C = \frac{q n_i^2}{G_B} e^{\frac{V_{BE}}{V_t}}$$

$$\frac{J_{Ep}}{J_C} = \frac{G_B}{G_E} \qquad \gamma = \frac{1}{1 + \frac{G_B}{G_E}} \qquad \beta_F = \frac{1}{\frac{1}{\gamma} - 1} = \frac{1}{1 + \frac{G_B}{G_E} - 1} = \frac{G_E}{G_B}$$

$$\frac{G_E}{G_B} = \frac{\int_0^{x_E'} \frac{N_E}{D_E} dx'}{\int^{x_B} \frac{N_B}{D_B} dx} = \frac{D_B}{D_E} \frac{\int_{0'}^{x_E'} N_E dx'}{\int^{x_B} N_B dx} = \frac{D_B}{D_E} \frac{Q_E}{Q_B} = 200$$

8.1

Wednesday, April 6, 2022 10:35 AM

НАПОН ПРАГА

$$V_T = \phi'_{ms} \pm |\phi_{si}| - \frac{Q_{ox}}{C_{ox}} \pm \frac{|Q_B|}{C_{ox}}$$

\+ NMOS (подлога p)

− PMOS (подлога n)

Si са Al g.

① $\phi'_{ms} = -0{,}6\,V \mp |\phi_f|$ — $-$ NMOS, $+$ PMOS

$q|\phi_f| = |E_i - E_{Fs}|$

$$|\phi_f| = V_t \ln \frac{N}{n_i}$$

② $|\phi_{si}| = 2|\phi_f| + 6V_t$

$q|\phi_{si}| = |E_i(+\infty) - E_i(0)|$

③ $Q_{ox} = qQ_{ss}$

$$C_{ox} = \frac{\varepsilon_{ox}\varepsilon_0}{x_{ox}}$$

④ $\pm |Q_B| / C_{ox}$

$Q_B = \mp qNx_d$ — $-$ NMOS, $+$ PMOS

$$x_d = \sqrt{\frac{2\varepsilon_s\varepsilon_0(|\phi_{si}| + V_{SB})}{qN}}$$

$$Q_B = \mp\sqrt{2qN\varepsilon_s\varepsilon_0(|\phi_{si}| + V_{SB})}$$

$V_{SB} = 0$

$$V_T = \underbrace{\phi'_{ms} - \frac{qQ_{ss}}{C_{ox}}}_{V_{FB}} \pm |\phi_{si}| \pm \frac{\sqrt{2\varepsilon_s\varepsilon_0 qN|\phi_{si}|}}{C_{ox}}$$

$V_T = V_{FB} \pm |\phi_{si}| \pm |Q_B|$

V_{FB} $\quad V_T = V_{FB} \pm |\varphi_{si}| \pm \dfrac{\cdot}{C_{ox}}$

8.1. $N_A = 5 \cdot 10^{14}\ cm^{-3}$ $\quad V_t = 25{,}85215\ mV$

$|\phi_{si}| = 2|\phi_f| + 6V_t$ $\quad n_i = 10^{10}\ cm^{-3}$

$|\phi_f| = V_t \ln \dfrac{N_A}{n_i}$ $\quad |\phi_f| = 279{,}7145\ mV = 0{,}28\ V$

$|\phi_{si}| = 714{,}542\ mV = 0{,}715\ V$

(б) $x_d, Q_B = ?$

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho}{\varepsilon_s\varepsilon_0} = \frac{qN_A}{\varepsilon_s\varepsilon_0}$$

$$\frac{d}{dx}\left(\frac{d\varphi}{dx}\right) = \frac{qN_A}{\varepsilon_s\varepsilon_0}$$

$$\int_{x_d}^{x} d\left(\frac{d\varphi}{dx}\right) = \int_{x_d}^{x} \frac{qN_A}{\varepsilon_s\varepsilon_0}\,dx$$

$$\frac{d\varphi}{dx} - \underbrace{\left.\frac{d\varphi}{dx}\right|_{x_d}}_{0} = \frac{qN_A}{\varepsilon_s\varepsilon_0}(x - x_d)$$

$$\frac{d\varphi}{dx} = \frac{qN_A}{\varepsilon_s\varepsilon_0}(x - x_d)$$

$$\int_{x_d}^{x} d\varphi = \frac{qN_A}{\varepsilon_s\varepsilon_0}\int_{x_d}^{x}(x - x_d)\,dx$$

$$\varphi(x) - \varphi(x_d) = \frac{qN_A}{2\varepsilon_s\varepsilon_0}(x - x_d)^2 \qquad \varphi(x_d) = 0$$

$$\varphi(x) = \frac{qN_A}{2\varepsilon_s\varepsilon_0}(x - x_d)^2$$

$$\phi_{si} = \varphi(0) = \frac{qN_A}{2\varepsilon_s\varepsilon_0}x_d^2$$

$$x_d = \sqrt{\frac{2\varepsilon_s\varepsilon_0|\phi_{si}|}{qN_A}}$$

$\varepsilon_s = 11{,}8$

$\varepsilon_0 = 8{,}8542 \cdot 10^{-14}\ \dfrac{F}{cm}$

$$q = 1{,}6022 \cdot 10^{-19}\ C$$

$$x_d = 1{,}365 \cdot 10^{-4}\ cm = 1{,}37\ \mu m$$

$$Q_B = -qN_A x_d = -1{,}0937 \cdot 10^{-8}\ \frac{C}{cm^2}$$

8.2

Wednesday, April 6, 2022 11:10 AM

$n-Si \quad N_D = 10^{15}\, cm^{-3} \Rightarrow$ PMOS Al-gate

$x_{ox} = 100\, nm \quad Q_{SS} = 2 \cdot 10^{11}\, cm^{-2} \quad V_T = ? \qquad 6V_t\,!$

$$V_T = \phi'_{ms} - |\phi_{si}| - \frac{qQ_{ss}}{C_{ox}} - \frac{|Q_B|}{C_{ox}}$$

① $\phi'_{ms} = -0{,}6V + |\phi_f| \qquad |\phi_f| = V_t \ln\frac{N_D}{n_i} = 297{,}6339\, mV = 0{,}298V$

$\phi'_{ms} = -0{,}302V$

② $|\phi_{si}| = 2|\phi_f| + 6V_t = 750{,}3807\, mV = 0{,}750V$

③ $C_{ox} = \frac{\varepsilon_{ox}\varepsilon_0}{x_{ox}} = 3{,}45 \cdot 10^{-8}\,\frac{F}{cm} \qquad \varepsilon_{ox} = 3{,}9$

$-\frac{qQ_{ss}}{C_{ox}} = -0{,}928V$

④ $|Q_B| = qN_D x_d = qN_D\sqrt{\frac{2\varepsilon_s\varepsilon_0|\phi_{si}|}{qN_D}} = \sqrt{2\varepsilon_s\varepsilon_0 qN_D|\phi_{si}|}$

$$|Q_B| = 1{,}585 \cdot 10 \quad \frac{}{cm^2}$$

$$\frac{|Q_B|}{C_{ox}} = 0{,}4594\,V$$

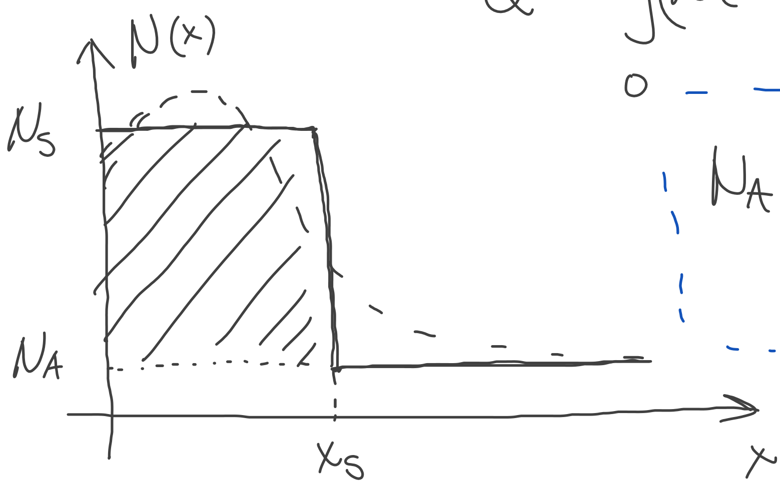
$$V_T = -0{,}302 - 0{,}75 - 0{,}928 - 0{,}459$$

$$V_T = -2{,}439\,V = -2{,}44\,V$$

PMOS са ИНДУКОВАНИМ КАНАЛОМ

8.13

Wednesday, April 6, 2022 11:11 AM



$$Q = \int_0^{+\infty} (N(x) - N_A)\,dx = (N_S - N_A)\,x_S$$

$V_{SB} = 2V$

$N_A = 7{,}5 \cdot 10^{15}\ cm^{-3}$

$x_S = 0{,}2\ \mu m \qquad N_S = 3 \cdot 10^{16}\ cm^{-3}$

$Q = 4{,}5 \cdot 10^{11}\ cm^{-2}$

$x_{ox} = 35\ nm$

$Q_{SS} = 10^{11}\ cm^{-2}$

$\Delta V_T = ?$

1) $x_d < x_S$

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho}{\varepsilon} = \frac{qN_S}{\varepsilon_0\varepsilon_S}$$

8.1 $\rightarrow \quad \left.\frac{d\varphi}{dx}\right|_{x_d} = 0 \quad , \quad \varphi(x_d) = 0$

$$\varphi(x) = \frac{qN_S}{2\varepsilon_0\varepsilon_S}(x - x_d)^2$$

$$\varphi(0) = |\phi_{Si}| + V_{SB} = \frac{qN_S x_d^2}{2\varepsilon_S\varepsilon_0}$$

$$x_d = \sqrt{\frac{2\varepsilon_S\varepsilon_0}{qN_S}(|\phi_{Si}| + V_{SB})}$$

$$|\phi_{Si}| = |\phi_{fS}| + |\phi_f|$$

$$|\phi_{fS}| = V_t \ln\frac{N_S}{n_i} \qquad |\phi_f| = V_t \ln\frac{N_A}{n_i}$$

$$|\phi_{Si}| = V_t \ln\frac{N_A N_S}{n_i^2} = 0{,}735\ V$$

$$x_d = 3{,}4507 \cdot 10^{-5}\ cm = 0{,}345\ \mu m > x_S = 0{,}2\ \mu m$$

2) $x_d > x_S$

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho}{\varepsilon} = \frac{qN(x)}{\ldots} \quad \Big/ \int^x (\quad)\,dx \qquad \left.\frac{d\varphi}{dx}\right| = 0$$

$$\frac{d\varphi}{dx} = \frac{qN_A}{\varepsilon_0\varepsilon_S}(x-x_d) \; ; \quad x_S \le x \le x_d$$

$$\frac{d\varphi}{dx} = \int_{x_d}^{x_S} \frac{qN_A}{\varepsilon_0\varepsilon_S}dx + \int_{x_S}^{x} \frac{qN_S}{\varepsilon_0\varepsilon_S}dx = \frac{qN_A}{\varepsilon_0\varepsilon_S}(x_S-x_d) + \frac{qN_S}{\varepsilon_0\varepsilon_S}(x-x_S) \; ; \quad 0 \le x \le x_S$$

$$\varphi(0) - \underbrace{\varphi(x_d)}_{=0} = \int_{x_d}^{x_S} \frac{qN_A}{\varepsilon_0\varepsilon_S}(x-x_d)dx + \int_{x_S}^{0} \frac{qN_S}{\varepsilon_0\varepsilon_S}(x-x_S)dx + \int_{x_S}^{0} \frac{qN_A}{\varepsilon_0\varepsilon_S}(x_S-x_d)dx$$

$$\varphi(0) = \frac{qN_A}{2\varepsilon_0\varepsilon_S}(x-x_d)\Big|_{x_d}^{x_S} + \frac{qN_S}{2\varepsilon_0\varepsilon_S}(x-x_S)^2\Big|_{x_S}^{0} + \frac{qN_A}{\varepsilon_0\varepsilon_S}(x_S-x_d)x\Big|_{x_S}^{0}$$

$$\varphi(0) = \frac{qN_A}{2\varepsilon_0\varepsilon_S}(x_S-x_d)^2 + \frac{qN_S}{2\varepsilon_0\varepsilon_S}x_S^2 + \frac{qN_A}{\varepsilon_0\varepsilon_S}x_d x_S - \frac{qN_A}{\varepsilon_0\varepsilon_S}x_S^2$$

$$\varphi(0) = \frac{qN_A}{2\varepsilon_0\varepsilon_S}x_S^2 + \frac{qN_A}{2\varepsilon_0\varepsilon_S}x_d^2 - \frac{qN_A}{\varepsilon_0\varepsilon_S}x_S x_d + \frac{qN_S}{2\varepsilon_0\varepsilon_S}x_S^2$$

$$- \frac{qN_A}{\varepsilon_0\varepsilon_S}x_S^2 \qquad + \frac{qN_A}{\varepsilon_0\varepsilon_S}x_S x_d$$

$$(N_S - N_A)x_S = Q$$

$$\varphi(0) = \frac{qN_A}{2\varepsilon_0\varepsilon_S}x_d^2 - \frac{qN_A}{2\varepsilon_0\varepsilon_S}x_S^2 + \frac{qN_S}{2\varepsilon_0\varepsilon_S}x_S^2 = \frac{qN_A}{2\varepsilon_0\varepsilon_S}x_d^2 + \frac{q}{2\varepsilon_0\varepsilon_S}(N_S-N_A)x_S^2$$

$$|\phi_{si}| + V_{SB} = \frac{qN_A x_d^2}{2\varepsilon_0\varepsilon_s} + \frac{qQx_s}{2\varepsilon_0\varepsilon_s}$$

$$x_d = \sqrt{\frac{2\varepsilon_s\varepsilon_0}{qN_A}\left(|\phi_{si}| + V_{SB} - \frac{qQx_s}{2\varepsilon_0\varepsilon_s}\right)} = 5{,}9691 \cdot 10^{-5}\ \text{cm} = 0{,}597\ \mu\text{m} > x_s = 0{,}2\ \mu\text{m}$$

$$V_T = V_{FB} + |\phi_{si}| + \frac{|Q_B|}{C_{ox}}$$

$$V_{FB} = \phi'_{ms} - \frac{qQ_{ss}}{C_{ox}} \qquad C_{ox} = \frac{\varepsilon_{ox}\varepsilon_0}{x_{ox}} = 9{,}87 \cdot 10^{-8}\ \frac{F}{cm}$$

$$\phi'_{ms} = -0{,}6V - |\phi_f|$$

$$V_{FB} = -1{,}11\ V$$

$$|\phi_f| = V_t \ln\frac{N_A}{n_i} = 0{,}35\ V \qquad \phi'_{ms} = -0{,}95\ V$$

$$|\phi_{si}| = 0{,}735\ V$$

$$\frac{qQ_{ss}}{C_{ox}} = 0{,}16\ V$$

$$Q_B = -qN_A x_d - q(N_s - N_A)x_s = -qN_A x_d - qQ = -1{,}44 \cdot 10^{-7}\ \frac{C}{cm^2}$$

$V_T = 1{,}08\ V$ после им.

$$V_T = V_{FB} + |\phi_{si}| + \frac{|Q_B|}{C_{ox}}$$

$$|\phi_{si}| = 2|\phi_f| = 2V_t \ln \frac{N_A}{n_i}$$

$$x_d = \sqrt{\frac{2\varepsilon_0\varepsilon_s}{qN_A}(|\phi_{si}| + V_{SB})}$$

$$Q_B = -qN_A x_d$$

$V_T = 0{,}4215\ V$ БЕЗ ИМПЛАНТАЦИЈЕ

(б) $x_s = 0{,}4\ \mu m$

$$x_d = 0{,}345\ \mu m < x_s$$

$$V_T = V_{FB} + |\phi_{si}| + \frac{|Q_B|}{C_{ox}}$$

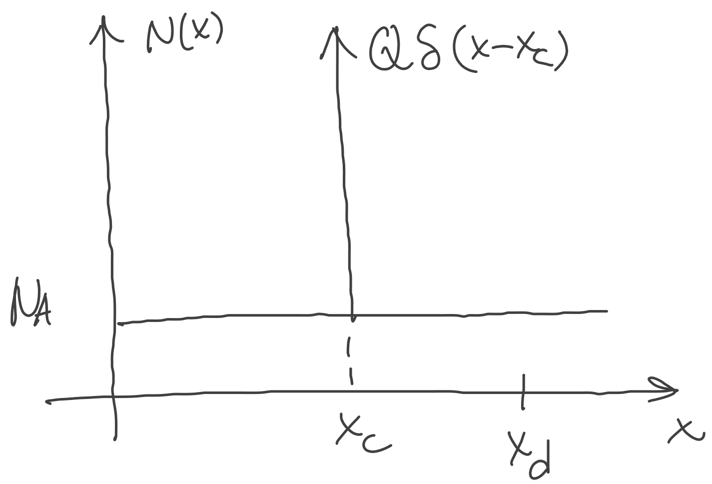
$V_{FB} = -1{,}11\ V$ $|\phi_{si}| = 0{,}735\ V$

$$|Q_B| = qN_s x_d$$

$$V_T = 1{,}2912\ V = 1{,}3\ V$$

8.15

Wednesday, April 6, 2022 12:04 PM



$$N(x) = N_A + Q\delta(x - x_c)$$

$$x_d > x_c$$

(a) ΔV_T ИЗРАЗ

(б) $\Delta V_T = ?$ $Q = 10^{11}\,\text{cm}^{-2}$

$x_c = 0$

$x_{ox} = 40\,\text{nm}$

$$\frac{d^2\varphi}{dx^2} = -\frac{\rho}{\varepsilon} = \frac{q}{\varepsilon_s\varepsilon_0}\left(N_A + Q\delta(x - x_c)\right) \quad \Big/ \int_{x_d}^{x} (\quad)\,dx$$

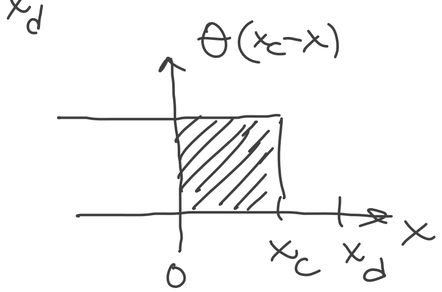
$$\frac{d\varphi}{dx} - \cancelto{0}{\frac{d\varphi}{dx}\bigg|_{x_d}} = \frac{qN_A}{\varepsilon_s\varepsilon_0}(x - x_d), \quad x > x_c$$

$$\frac{d\varphi}{dx} = \frac{qN_A}{\varepsilon_s\varepsilon_0}(x - x_d) + \frac{qQ}{\varepsilon_0\varepsilon_s}\underbrace{\int_{x_d}^{x}\delta(x - x_c)\,dx}_{-1}$$

jer je $x < x_c < x_d$

$$\frac{d\varphi}{dx} = \frac{qN_A}{\varepsilon_0\varepsilon_s}(x - x_d) - \frac{qQ}{\varepsilon_0\varepsilon_s}, \quad x < x_c$$

$$\frac{d\varphi}{dx} = \frac{qN_A}{\varepsilon_0\varepsilon_s}(x - x_d) - \frac{qQ}{\varepsilon_0\varepsilon_s}\theta(x_c - x) \quad \Big/ \int_{x_d}^{0} (\quad)\,dx$$



$$\varphi(0) - \cancelto{0}{\varphi(x_d)} = \frac{qN_A}{2\varepsilon_0\varepsilon_s}(x - x_d)^2\bigg|_{x_d}^{0} - \frac{qQ}{\varepsilon_0\varepsilon_s}(0 - x_c)$$

$$|\Phi_{si}| = \varphi(0) = \frac{qN_A}{2\varepsilon_0\varepsilon_s}x_d^2 + \frac{qQ}{\varepsilon_0\varepsilon_s}x_c$$

$$x_d = \sqrt{\frac{2\varepsilon_0\varepsilon_s}{qN_A}\left(|\Phi_{si}| - \frac{qQx_c}{\varepsilon_0\varepsilon_s}\right)}$$

$$V_T = V_{FB} + |\phi_{Si}| + \frac{|Q_B|}{C_{ox}}$$

$$\to |Q_B| = qN_A x_{d0} = \sqrt{2\varepsilon_0\varepsilon_S qN_A|\phi_{Si}|}$$

$$\to |Q_B'| = qN_A x_d + qQ$$

$$\Delta V_T = \frac{|Q_B'|}{C_{ox}} - \frac{|Q_B|}{C_{ox}} = \frac{1}{C_{ox}}\left(\sqrt{2\varepsilon_S\varepsilon_0 qN_A\left(|\phi_{Si}| - \frac{qQ}{\varepsilon_S\varepsilon_0}x_c\right)} + qQ - \sqrt{2\varepsilon_0\varepsilon_S qN_A|\phi_{Si}|}\right)$$

$$\Delta V_T(x_c = 0) = \frac{qQ}{C_{ox}}$$

(e) $\Delta V_T = 0{,}186\,V$

8.21 I 8.22

Wednesday, April 6, 2022 12:30 PM

V_p $\quad x_d = \sqrt{\frac{2\varepsilon_0\varepsilon_s}{qN_A}(V_{DSsat} + V_p)}$

$$x_d = \sqrt{\frac{2\varepsilon_0\varepsilon_s}{qN_A}(V_{DSsat} + \Delta V + V_p)}$$

$$\Delta L = L - L' = x_d(V_{DSsat} + \Delta V) - x_d(V_{DSsat})$$

$$\Delta L = \sqrt{\frac{2\varepsilon_s\varepsilon_0}{qN_A}}\left(\sqrt{V_{DSsat} + \Delta V + V_p} - \sqrt{V_{DSsat} + V_p}\right)$$

$$I_{Dsat} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{GS} - V_T)^2$$

$$I_{Dsat}(\Delta V) = \frac{1}{2}\mu C_{ox}\frac{W}{L - \Delta L}(V_{GS} - V_T)^2$$

$$\frac{I_{Dsat}(\Delta V)}{I_{Dsat}} = \frac{L}{L - \Delta L}$$

8.22 $\Delta V = 0{,}1\,V$, $L = 5\,\mu m$, $N_A = 10^{15}\,cm^{-3}$, $V_{DSsat} = 3V$, $V_p = 1V$

$\Delta L = 0{,}0284\,\mu m$

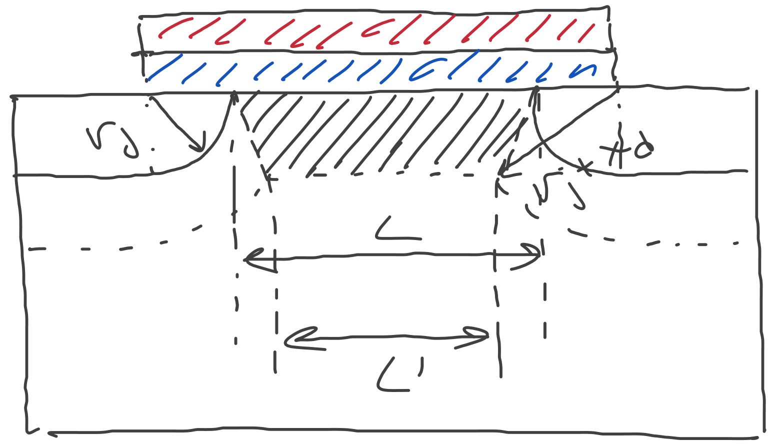
$$\frac{I_{Dsat}(\Delta V)}{I_{Dsat}} = 1{,}0057 \quad \nearrow 6‰$$

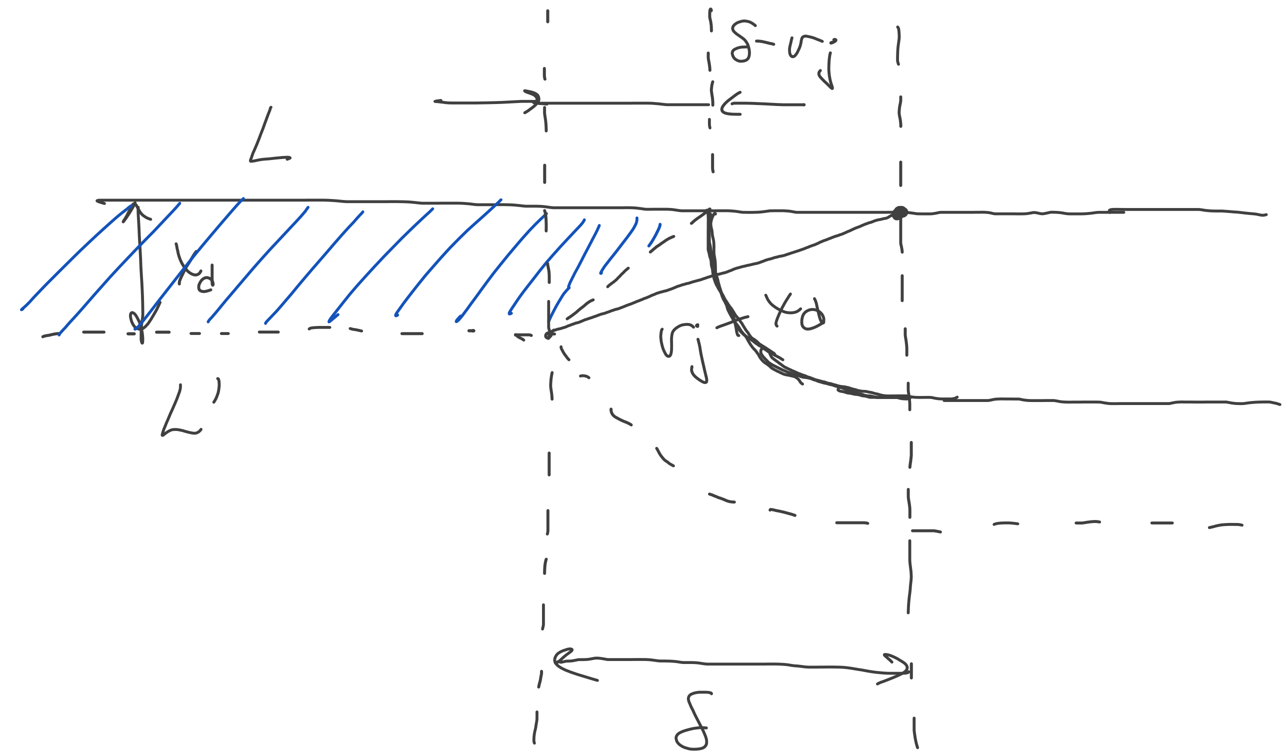
(б) $L = 0{,}5\ \mu m$ остали подаци исти

$$\frac{I_{Dsat}(\Delta V)}{I_{Dsat}} = 1{,}0602 \quad \nearrow 6\%$$

8.24

Wednesday, April 6, 2022 12:42 PM





$$L' = L - 2(\delta - r_j)$$

$$\delta = \sqrt{(r_j + x_d)^2 - x_d^2}$$

$$L + L' = L + L + 2r_j - 2\delta$$

$$L + L' = 2L + 2r_j - 2\sqrt{(r_j + x_d)^2 - x_d^2}$$

$$L + L' = 2L + 2r_j - 2r_j\sqrt{1 + \frac{2x_d}{r_j}}$$

$$L + L' = 2L - 2r_j\left(\sqrt{1 + \frac{2x_d}{r_j}} - 1\right)$$

$$\frac{L + L'}{2L} = 1 - \underbrace{\frac{r_j}{L}\left(\sqrt{1 + \frac{2x_d}{r_j}} - 1\right)}_{n}$$

$$\frac{L + L'}{2L} = 1 - n$$

$$\overline{Q_B'} = -qN_A V = -qN_A x_d \frac{L + L'}{2} W$$

$$Q_B' = \frac{\overline{Q_B'}}{W} = -qN_A x_d \frac{L + L'}{}$$

$$LW \qquad w \qquad 2L$$

$$Q_B = -q N_A x_d$$

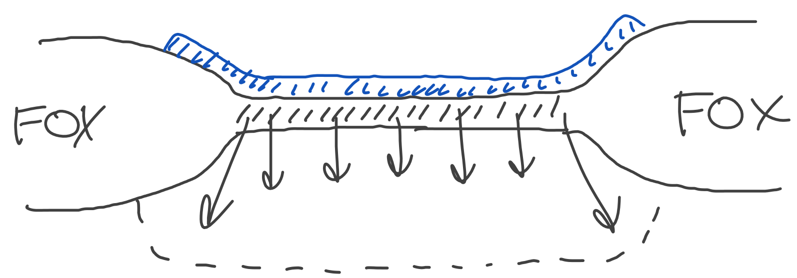
$$|Q_B'| - |Q_B| = q N_A x_d \left(\frac{L+L'}{2L} - 1 \right) = -q N_A x_d n = -|Q_B| n$$

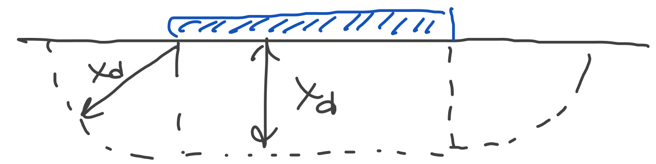
$$\Delta V_T = -\frac{|Q_B|}{C_{ox}} \frac{r_j}{L} \left(\sqrt{1 + \frac{2x_d}{r_j}} - 1 \right)$$

$$V_T = V_{FB} + |\phi_{si}| + \frac{|Q_B|}{C_{ox}} \left[1 - \frac{r_j}{L} \left(\sqrt{1 + \frac{2x_d}{r_j}} - 1 \right) \right]$$

8.25

Wednesday, April 6, 2022 1:08 PM





$$|\overline{Q_B'}| = qN_A x_d WL + 2\, qN_A \frac{x_d^2 \pi}{4} L \quad / \; WL$$

$$|Q_B'| = \underbrace{qN_A x_d}_{|Q_B|} + \underbrace{\frac{qN_A x_d^2 \pi}{2W}}_{|\Delta Q_B|}$$

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$$\Delta V_T = \frac{|\Delta Q_B|}{C_{ox}} = \frac{qN_A x_d^2 \pi}{2WC_{ox}}$$

8.26

Wednesday, April 6, 2022 1:02 PM

$N_A = 2\cdot 10^{16}\ cm^{-3}$

$x_{ox} = 30nm$

$r_j = 0{,}2\ \mu m$

$V_{FB} = -1V$

$L = 1\mu m$

$W = 15\mu m$

ПО МОДЕЛУ ДУГОКАНАЛНОГ

$$V_T = V_{FB} + |\Phi_{Si}| + \frac{|Q_B|}{C_{ox}}$$

$$|\Phi_{Si}| = 2V_t \ln\frac{N_A}{n_i} + 6V_t = 0{,}9053V$$

$$x_d = \sqrt{\frac{2\varepsilon_0\varepsilon_S|\Phi_{Si}|}{qN_A}} = 0{,}243\mu m$$

$$|Q_B| = qN_A x_d = 7{,}7856\cdot 10^{-8}\ \frac{C}{cm^2}$$

$$C_{ox} = 1{,}151\cdot 10^{-7}\ \frac{F}{cm^2}$$

$$V_T = 0{,}5817V$$

ПО МОДЕЛУ ПОДЕЉЕНОГ Н.

$$V_T = V_{FB} + |\Phi_{Si}| + \frac{|Q_B|}{C_{ox}}\underbrace{\left[1 - \frac{r_j}{L}\left(\sqrt{1+\frac{2x_d}{r_j}} - 1\right)\right]}_{0{,}8296}$$

$$V_T = 0{,}4664\ V$$

8.29

Wednesday, April 20, 2022 9:15 AM

Al - gate $N_D = 10^{15}\,cm^{-3}$

$x_{ox} = 100\,nm$ $Q_{ss} = 2\cdot 10^{11}\,cm^{-2}$

(a) CE , $\lambda = 2$, $\lambda = 10$

(b) CV , $\lambda = 2$, $\lambda = 10$ $k = \sqrt{\lambda}$

$6V_t$!

$V_{SB} = 0$

(a) CE

W, L	x_{ox}	N	V
$\frac{1}{\lambda}$	$\frac{1}{\lambda}$	λ	$\frac{1}{\lambda}$

$$|\phi_f| = V_t \ln\left(\frac{\lambda N_D}{n_i}\right) = \begin{cases} 0{,}3156\,V ; & \lambda = 2 \\ 0{,}3572\,V ; & \lambda = 10 \end{cases}$$

$$|\phi_{si}| = 2|\phi_f| + 6V_t = \begin{cases} 0{,}7862\,V ; & \lambda = 2 \\ 0{,}8694\,V ; & \lambda = 10 \end{cases}$$

$$\phi'_{ms} = -0{,}6V + |\phi_f| = \begin{cases} -0{,}2844\,V ; & \lambda = 2 \\ -0{,}2428\,V ; & \lambda = 10 \end{cases}$$

$$C'_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{\frac{x_{ox}}{\lambda}} = \lambda \frac{\varepsilon_0 \varepsilon_{ox}}{x_{ox}} = \lambda C_{ox}$$

$$-\frac{qQ_{ss}}{\lambda C_{ox}} = \begin{cases} -0{,}464\,V ; & \lambda = 2 \\ -0{,}0928\,V ; & \lambda = 10 \end{cases}$$

$$|Q'_B| = \sqrt{2\varepsilon_s \varepsilon_0 \lambda N_D |\phi_{si}|} = \sqrt{\lambda}\,|Q_B|$$

$$\frac{|Q'_B|}{C'_{ox}} = \frac{\sqrt{\lambda}|Q_B|}{\lambda C_{ox}} = \frac{1}{\sqrt{\lambda}} \frac{|Q_B|}{C_{ox}} = \begin{cases} 0{,}3322\,V ; & \lambda = 2 \\ 0{,}1562\,V ; & \lambda = 10 \end{cases}$$

$$V'_T = \phi'_{ms} - \frac{qQ_{ss}}{\lambda C_{ox}} - |\phi_{si}| - \frac{1}{\sqrt{\lambda}} \frac{|Q_B|}{C_{ox}} = \begin{cases} -1{,}8669\,V ; & \lambda = 2 \\ \ldots \end{cases}$$

$$\begin{cases} -1{,}3613\,V ; \lambda = 10 \end{cases}$$

CV

W, L	x_{ox}	N	V
$\frac{1}{\lambda}$	$\frac{1}{K}$	λ	1

$$C'_{ox} = \frac{\varepsilon_0 \varepsilon_{ox}}{\frac{x_{ox}}{K}} = \sqrt{\lambda}\,\frac{\varepsilon_0 \varepsilon_{ox}}{x_{ox}} = \sqrt{\lambda}\,C_{ox}$$

$$-\frac{qQ_{ss}}{\sqrt{\lambda}\,C_{ox}} = \begin{cases} -0{,}6562\,V ; & \lambda = 2 \\ -0{,}2934\,V ; & \lambda = 10 \end{cases}$$

$$\frac{|Q'_B|}{C'_{ox}} = \frac{\sqrt{\lambda}\,|Q_B|}{\sqrt{\lambda}\,C_{ox}} = \begin{cases} 0{,}4698\,V ; & \lambda = 2 \\ 0{,}4941\,V ; & \lambda = 10 \end{cases}$$

$$V'_T = \begin{cases} -2{,}1967\,V ; & \lambda = 2 \\ -1{,}8998\,V ; & \lambda = 10 \end{cases}$$

8.30

Wednesday, April 20, 2022 9:39 AM

(a) $L = 1\,\mu m$ $\quad I_{Dsat} = ?$ $\quad$ CE $\quad \lambda = 2, 10$

$W = 30\,\mu m$

$\mu_p = 320\,\frac{cm^2}{Vs}$ $\quad V_{GS} = -8\,V$ $\quad$ (b) CV $\quad \lambda = 2, 10$ $\quad k = \sqrt{\lambda}$

$$I_{Dsat} = -K\,\frac{(V_{GS} - V_T)^2}{2} \qquad K = \mu_p C_{ox}\frac{W}{L} = 3{,}3150 \cdot 10^{-4}\,\frac{A}{V^2}$$

CE

$$K' = \mu_p\,\lambda C_{ox}\,\frac{\frac{1}{\lambda}W}{\frac{1}{\lambda}L} = \lambda K \qquad V_{GS}' = \frac{V_{GS}}{\lambda} = \begin{cases} -4V,\ \lambda = 2 & < V_T' = -1{,}87V \quad \text{ON} \\ -0{,}8V,\ \lambda = 10 & > V_T' = -1{,}36V \quad \text{OFF} \end{cases}$$

$$I_{Dsat}' = -\lambda K\,\frac{\left(\frac{V_{GS}}{\lambda} - V_T'\right)^2}{2} = \begin{cases} -1{,}5\,mA,\ \lambda = 2 \\ 0,\ \lambda = 10 \end{cases}$$

CV

$$x_{ox}' = \frac{x_{ox}}{k} = \frac{x_{ox}}{\sqrt{\lambda}} \Rightarrow C_{ox}' = \sqrt{\lambda}\,C_{ox} \qquad K' = \sqrt{\lambda}\,K$$

$$I_{Dsat}' = -K\sqrt{\lambda}\,\frac{(V_{GS} - V_T')^2}{2} = \begin{cases} -7{,}9\,mA;\ \lambda = 2 \\ -19{,}5\,mA;\ \lambda = 10 \end{cases}$$

10.1

Wednesday, April 27, 2022 9:50 AM

Print

$V(y) = V_{DS}\frac{y}{L}$

$V_d = V_{bi} + V(y) - V_{GS}$

$x_d(y) = \sqrt{\frac{2\varepsilon_s\varepsilon_0}{qN_D}V_d(y)}$

$V_{DS} = 0,\ V(y) = 0$

$V_{GS} = V_T \Rightarrow x_d = a$

$a = \sqrt{\frac{2\varepsilon_s\varepsilon_0}{qN_D}(V_{bi} - V_T)}$

$a^2 = \frac{2\varepsilon_s\varepsilon_0}{qN_D}(V_{bi} - V_T)$

$V_T = V_{bi} - \frac{qN_D a^2}{2\varepsilon_s\varepsilon_0}$

$V_T = V_{bi} - V_P$

$V_P = \frac{qN_D a^2}{2\varepsilon_s\varepsilon_0}$

НАПОН ДОДИРА

$V_{DS} \neq 0$

$V(L) = V_{DS\,sat},\ V_d(L) = V_P$

$V_d = V_{bi} + V(y) - V_{GS}$

$V_P = V_{bi} + V_{DS\,sat} - V_{GS}$

$V_{DS\,sat} = V_{GS} - V_{bi} + V_P$

$V_{DS\,sat} = V_{GS} - (V_{bi} - V_P)$

$V_{DS\,sat} = V_{GS} - V_T$

$dV = I_D dR$

$dV = \frac{dy}{\sigma W(a - x_d)} I_D$

$I_D \frac{dy}{\sigma W} = (a - x_d(V))dV$

$\sigma = const$

$I_D = const$

$x_d = \underbrace{\sqrt{\frac{2\varepsilon_s\varepsilon_0}{qN_D}}}_{\frac{a}{\sqrt{V_P}}}\sqrt{V(y) - V_{GS} + V_{bi}} = a\frac{\sqrt{V - V_{GS} + V_{bi}}}{\sqrt{V_P}}$

L V_{DS}

$$I_D \frac{1}{\sigma W}\int_0 dy = a\int_0^{V_{DS}}\left(1-\frac{\sqrt{V-V_{GS}+V_{bi}}}{\sqrt{V_p}}\right)dV$$

$$I_D = \frac{\sigma a W}{L}\left\{V_{DS}-\frac{2}{3}V_p\left[\left(\frac{V_{DS}-V_{GS}+V_{bi}}{V_p}\right)^{\frac{3}{2}}-\left(\frac{-V_{GS}+V_{bi}}{V_p}\right)^{\frac{3}{2}}\right]\right\}$$

$$\frac{\sigma a W}{L} = G_0 = q\mu_n N_D \frac{aW}{L}$$ ПРОВОДНОСТ за $x_d = 0$

$$V_{GS}^* = V_{GS}-V_{bi}$$

$$I_D = G_0\left\{V_{DS}-\frac{2}{3}V_p\left[\left(\frac{V_{DS}-V_{GS}^*}{V_p}\right)^{\frac{3}{2}}-\left(\frac{-V_{GS}^*}{V_p}\right)^{\frac{3}{2}}\right]\right\}$$

ЛИНЕАРНА ОБЛАСТ $V_{DS} << -V_{GS}^*$

$$I_D = G_0\left\{V_{DS}-\frac{2}{3}V_p\left(-\frac{V_{GS}^*}{V_p}\right)^{\frac{3}{2}}\left[\underbrace{\left(1-\frac{V_{DS}}{V_{GS}^*}\right)^{\frac{3}{2}}}-1\right]\right\}$$

$$I_D = G_0\left\{V_{DS}-\frac{2}{3}V_p\left(-\frac{V_{GS}^*}{V_p}\right)^{\frac{3}{2}}\left(1+\frac{3}{2}\frac{V_{DS}}{V_{GS}^*}-1\right)\right\}$$

$$I_D = G_0\left[V_{DS}-\sqrt{\frac{-V_{GS}^*}{V_p}}\,V_{DS}\right] = G_0 V_{DS}\left[1-\sqrt{\frac{-V_{GS}^*}{V_p}}\right]$$

ЗАСИЋЕЊЕ $V_{DSsat} = V_{GS} - V_T = V_{GS} - (V_{bi} - V_P) = V_{GS} - V_{bi} + V_P = V_{GS}^* + V_P$

Print

$$I_{Dsat} = G_0 \left\{ V_{GS}^* + V_P - \frac{2}{3} V_P \left[\left(\frac{V_P}{V_P}\right)^{\frac{3}{2}} - \left(\frac{-V_{GS}^*}{V_P}\right)^{\frac{3}{2}} \right] \right\}$$

$$I_{Dsat} = G_0 \left\{ V_{GS}^* + V_P - \frac{2}{3} V_P + \frac{2}{3} V_P \left(\frac{-V_{GS}^*}{V_P}\right)^{\frac{3}{2}} \right\}$$

$$I_{Dsat} = G_0 \left\{ \frac{V_P}{3} - (-V_{GS}^*) + \frac{2}{3} V_P \left(\frac{-V_{GS}^*}{V_P}\right)^{\frac{3}{2}} \right\}$$

$$I_{Dsat} = \frac{G_0 V_P}{3} \left[1 - 3 \frac{-V_{GS}^*}{V_P} + 2 \left(\frac{-V_{GS}^*}{V_P}\right)^{\frac{3}{2}} \right]$$

10.2

Wednesday, April 27, 2022 10:57 AM

$a = 0{,}2\ \mu m$

$N_D = 10^{16}\ cm^{-3}$

$\phi_B = 0{,}8\ V$

(a) $x_{d0} = \sqrt{\dfrac{2\varepsilon_s\varepsilon_0 V_{bi}}{q N_D}}$

$$\boxed{V_{bi} = \phi_B - V_t \ln\frac{N_c}{N_D}} = 0{,}7022\ V$$

$x_{d0} = 3{,}1885 \cdot 10^{-5}\ cm = 0{,}319\ \mu m$

$x_{d0} > a \Rightarrow$ EMESFET

$$\boxed{V_T = V_{bi} - V_p}$$

$$V_T = V_{bi} - \boxed{\frac{q N_D}{2\varepsilon_s\varepsilon_0} a^2} = 0{,}4259\ V > 0 \Rightarrow \text{EMESFET}$$

(b) $V_{ac} = ?$ $\quad x_{d1} = 0{,}15\ \mu m$

$$x_{d1} = \sqrt{\frac{2\varepsilon_s\varepsilon_0}{q N_D}\left(V_{bi} - V_{ac}\right)}$$

$$V_{ac} = V_{bi} - \frac{q N_D}{2\varepsilon_s\varepsilon_0} x_{d1}^2 = 0{,}5468\ V$$

10.3

Wednesday, April 27, 2022 10:46 AM

$\phi_B = 0{,}8\,V$

$\mu_n = 5000\ \frac{cm^2}{Vs}$

$W = 25\ \mu m$

$L = 1\ \mu m$

$a = 0{,}25\ \mu m$

$N_D = 10^{16}\ cm^{-3}$

$V_{GS} = 0{,}5\ V$

$$V_T = V_{bi} - V_P$$

$$V_{bi} = \phi_m - \phi_S = \phi_m - \left(\chi + \frac{E_C - E_F}{q}\right)$$

$$V_{bi} = \phi_B - \frac{E_C - E_F}{q}$$

$$n = N_D = N_C e^{\frac{E_F - E_C}{k_B T}}$$

$$\frac{E_C - E_F}{q} = V_t \ln\frac{N_C}{N_D}$$

$$V_{bi} = \phi_B - V_t \ln\frac{N_C}{N_D} \qquad N_C^{GaAs} = 4{,}4 \cdot 10^{17}\ cm^{-3}$$

$$V_{bi} = 0{,}7022\ V$$

$$V_P = \frac{qN_D}{2\varepsilon_r\varepsilon_0}a^2 = 0{,}4317\ V \qquad \varepsilon_S^{GaAs} = 13{,}1$$

$$V_T = V_{bi} - V_P = 0{,}2705\ V \qquad V_T > 0 \Rightarrow \text{EMESFET}$$

$$V_{DS\,sat} = V_{GS} - V_T = 0{,}2295\ V$$

$$I_{Dsat} = \frac{G_0 V_P}{3}\left[1 - 3\,\frac{-V_{GS}^*}{V_P} + 2\left(\frac{-V_{GS}^*}{V_P}\right)^{\frac{3}{2}}\right] \qquad G_0 = q\mu_n N_D \frac{aW}{L} = 5\,mS$$

$$I_{Dsat} = 1{,}18\,mA$$

$$V_{GS}^{*} = V_{GS} - V_{bi}$$

10.9

Wednesday, May 4, 2022 9:21 AM

$$\Phi_B = 0{,}950\ \text{V}$$

$$a = 0{,}5\ \mu\text{m}$$

$$N_D = 10^{16}\ \text{cm}^{-3}$$

$$V_T = V_{bi} - V_P$$

$$V_P = \frac{qN_D}{2\varepsilon_s\varepsilon_0}a^2 = 1{,}7266\ \text{V}$$

$$V_{bi} = \phi_m - \phi_s$$

$$V_{bi} = \phi_m - \left(\chi + \frac{E_c - E_F}{q}\right)$$

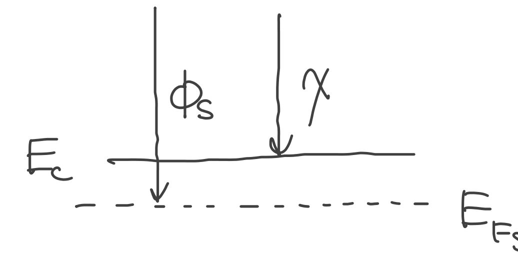
$$= \underbrace{\phi_m - \chi}_{\phi_B} - \frac{E_c - E_F}{q} = \phi_B - \frac{E_c - E_F}{q}$$

$$N_D = N_C\, e^{\frac{E_F - E_c}{k_B T}}$$

$$\ln\frac{N_D}{N_C} = -(E_c - E_F)/k_B T$$

$$E_c - E_F = k_B T \ln\frac{N_c}{N_D}$$

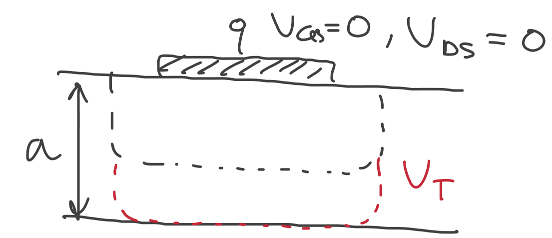
$$\frac{E_c - E_F}{q} = V_t \ln\frac{N_c}{N_D}$$



$$V_{bi} = \phi_B - V_t \ln\frac{N_c}{N_D} = 0{,}8522\ \text{V}$$

$$V_T = V_{bi} - V_P = -0{,}8745\ \text{V}$$

$\Rightarrow$ DMESFET



$$x_{d0} = \sqrt{\frac{2\varepsilon_s\varepsilon_0}{qN_D}V_{bi}} = 0{,}3513\ \mu\text{m} < a$$

$\Rightarrow$ DMESFET

Heterostrukture 1 – GaAs rešetka

Wednesday, April 20, 2022 10:49 AM

$\{hkl\}$ скуп еквивалентне равни (hkl)

кубна решетка

$\langle hkl \rangle$ скуп еквивалентних праваца $[hkl]$

$[hkl] \perp (hkl)$

$\langle 1\,0\,0 \rangle \to [1\,0\,0],\ [0\,1\,0],\ [0\,0\,1]$

$\langle 1\,0\,0 \rangle \to \frac{a}{2}$

$\langle 1\,1\,0 \rangle \to \frac{a\sqrt{2}}{4}$

$\langle 1\,1\,1 \rangle \to \frac{a}{\sqrt{3}}$

MBE 1

Wednesday, April 27, 2022 9:03 AM

$$\boxed{F = \frac{1}{4} n\bar{c}} \qquad \bar{c} = v_{sr} \qquad \bar{c} = \sqrt{\frac{8}{3\pi}}\, c_{rms} \qquad c_{rms} = v_{eff} = \sqrt{\frac{3k_BT}{m}}$$

$$\boxed{p = n k_B T} \qquad \boxed{\bar{c} = \sqrt{\frac{8k_BT}{\pi m}}} \qquad \frac{1}{2} m c_{rms}^2 = \frac{3}{2} k_B T$$

$$F = \frac{1}{4} n\bar{c} = \frac{1}{4} \frac{p}{k_BT} \sqrt{\frac{8}{3\pi}} \sqrt{\frac{3k_BT}{m}} = \boxed{\frac{p}{\sqrt{2\pi m k_B T}}}$$

$$p = 5 \cdot 10^{-11}\ mbar = 5 \cdot 10^{-11} \cdot 10^{-3}\, 10^{5}\ Pa = 5 \cdot 10^{-9}\ Pa$$

$$m = \frac{M_{H_2}}{N_A} = \frac{2{,}01588 \cdot 10^{-3}\ kg/mol}{6{,}02214 \cdot 10^{23}\ \frac{1}{mol}} \qquad k_B = 1{,}381 \cdot 10^{-23}\ \frac{J}{K}$$

$$F_{bg} = 5{,}3563 \cdot 10^{14}\ \frac{1}{m^2 s} = 5{,}4 \cdot 10^{14}\ \frac{1}{m^2 s}$$

GaAs

$$a = 0{,}5653\ nm \rightarrow (0\ 0\ 1) \longrightarrow \bar{N} = \frac{2}{a^2}$$

$$t_{bg} = \frac{\bar{N}}{\xi F_{bg}} = \frac{2}{\xi a^2 F_{bg}} = 1{,}2 \cdot 10^{4}\ s \approx 3h$$

ВРЕМЕ ДА СЕ НАРАСТЕ МОНОСЛОЈ H_2

= ВРЕМЕ ТОКОМ КОГ ЈЕ ПОВРШИНА

ЧИСТА ОД П. ГАСА

MBE 2

Wednesday, April 27, 2022 9:41 AM

$$F_K = \frac{p_K}{\sqrt{2\pi m_{Ga} k_B T_K}}$$ ФЛУКС НА ОТВОРУ К-ЋЕЛИЈЕ

$$\frac{\left[\frac{1}{m^2}\right]}{\left[\frac{1}{m^2 s}\right]} \to [s]$$

$\pi r^2 F_K$ БРЗИНА КОЈОМ Ga НАПУШТАЈУ К-ЋЕЛИЈУ

$$\frac{\pi r^2 F_K}{2\pi R^2} = \frac{1}{2}\left(\frac{r}{R}\right)^2 F_K$$ ФЛУКС Ga НА ПОЛУСФЕРУ

$$F_{Ga} = 2 \cdot \frac{1}{2}\left(\frac{r}{R}\right)^2 F_K$$ ФЛУКС Ga НА ПЛОЧИЦУ

$$t_{ML} = \frac{\bar{N}}{\xi F_{Ga}} = 2\left(\frac{R}{ar}\right)^2 \frac{\sqrt{2\pi m_{Ga} k_B T_K}}{p_K} \approx 5s$$

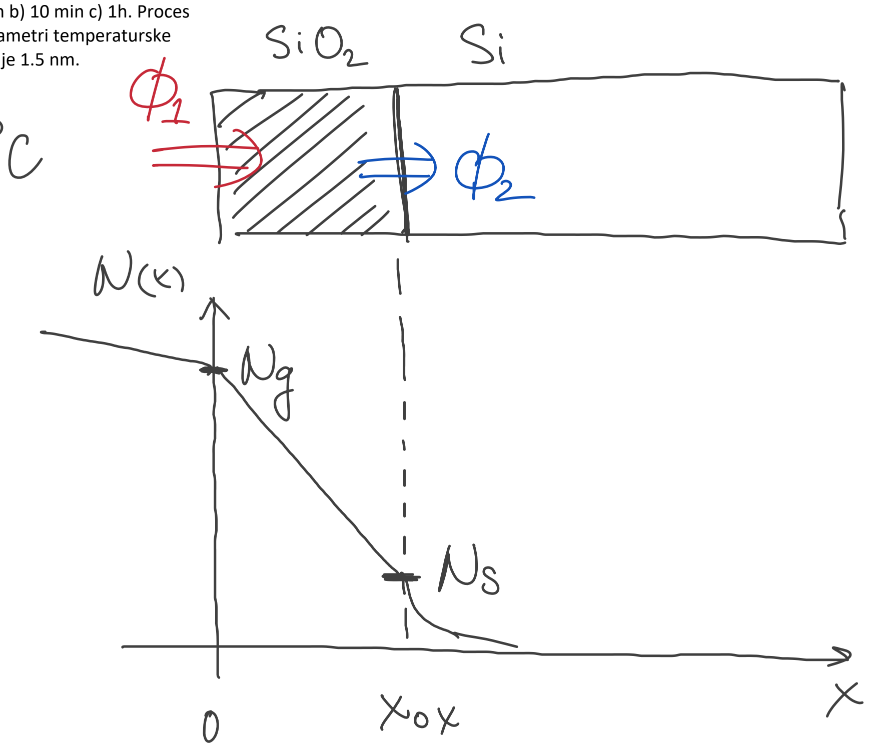
$$\frac{t_{ML}}{t_{bg}} = 5 \cdot 10^{-4} \gg 10^{-6}$$ $\xi_{H_2} = 1$

Oksidacija

Wednesday, March 16, 2022 10:43 AM

Deal - Grove

Izračunati debljinu oksidnog sloja, naraslog na silicijumskoj podlozi posle a) 1 min b) 10 min c) 1h. Proces oksidacije se odvija u vlažnoj atmosferi kiseonika na 1050 step. C. Poznati su parametri temperaturske zavisnosti Deal-Grove-ovih koeficijenata, dati u tabeli. Debljina prirodnog oksida je 1.5 nm.



$x_{ox} = ?$ (a) 1 min (b) 10 min (c) 1 h $t = 1050\,^\circ C$

$C e^{-\frac{E_a}{k_B T}}$

	B	B/A
C	$2{,}14 \cdot 10^{2}\ \mu m/h$	$8{,}95 \cdot 10^{7}\ \mu m/h$
E	$0{,}71\ eV$	$2{,}05\ eV$

$X_0 = 1{,}5\ nm$

$$\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial x^2} = 0 \Rightarrow \frac{dN}{dx} = a \Rightarrow N(x) = ax + b$$

$$N(0) = b = N_g \qquad N(x_{ox}) = N_s = a x_{xo} + N_g$$

$$a = \frac{N_s - N_g}{x_{ox}} = -\frac{N_g - N_s}{x_{ox}}$$

$$N(x) = N_g - \frac{N_g - N_s}{x_{ox}} x$$

$$\Phi_1 = -D \frac{dN}{dx} = D \frac{N_g - N_s}{x_{ox}} \qquad \Phi_2 = k(N_s - N_e) \qquad k = k(T) \sim e^{-\frac{E}{k_B T}}$$

$$\Phi_1 = \Phi_2 = \Phi$$

$$k(N_s - N_e) = D \frac{N_g - N_s}{x_{ox}} \qquad Si + O_2 \rightarrow SiO_2$$

$$\boxed{N_0 = \frac{dx_{ox}}{\ldots} = \Phi} \qquad \Phi \Delta t A = N_{ox} \Delta V_{\ldots} = N_{\ldots} A \Delta x_{\ldots}$$

$$\boxed{v = \frac{dx_{ox}}{dt} = \frac{\Phi}{N_{ox}}} \qquad \Phi \Delta t A = N_{ox} \Delta V_{ox} = N_{ox} A \Delta x_{ox}$$

$$\frac{\Delta x_{ox}}{\Delta t} = \frac{\Phi}{N_{ox}}$$

$$\underset{\sim\sim\sim}{N_g - N_e} = N_g - N_s + N_s - N_e = N_g - N_s + \frac{D}{k x_{ox}}(N_g - N_s)$$

$$N_g - N_e = (N_g - N_s)\left(1 + \frac{D}{k x_{ox}}\right)$$

$$N_g - N_s = \frac{N_g - N_e}{1 + \frac{D}{k x_{ox}}}$$

$$\Phi = \frac{D}{x_{ox}}(N_g - N_s) = \frac{D}{x_{ox}} \frac{N_g - N_e}{1 + \frac{D}{k x_{ox}}}$$

$$v = \frac{\Phi}{N_{ox}} = \frac{D}{N_{ox} x_{ox}} \frac{N_g - N_e}{1 + \frac{D}{k x_{ox}}}$$

$$\boxed{\frac{dx_{ox}}{dt} = \frac{D}{N_{ox}} \frac{N_g - N_e}{x_{ox} + \frac{D}{k}}}$$

$$\left(x_{ox} + \frac{D}{k}\right) dx_{ox} = \frac{D}{N_{ox}}(N_g - N_e)\, dt$$

$$\frac{x_{ox}^2}{2} + \frac{D}{k} x_{ox} = \frac{D(N_g - N_e)}{N_{ox}} t + C_1 \quad /2$$

N_{ox}

$$x_{ox}^2 + \underbrace{\frac{2D}{k}}_{A} x_{ox} = \underbrace{\frac{2D(N_g - N_e)}{N_{ox}}}_{B} t + \underbrace{C}_{B\tau}$$

$$x_{ox}^2 + A x_{ox} = B(t+\tau) \qquad x_{ox}(0) = X_0 \qquad x_0^2 + A x_0 = B\tau \Rightarrow \tau = \frac{x_0^2 + A x_0}{B}$$

$$x_{ox}^2 + A x_{ox} - B(t+\tau) = 0$$

$$x_{ox} = -\frac{A}{2} \oplus \sqrt{\frac{A^2}{4} + B(t+\tau)} = -\frac{A}{2} + \frac{1}{2}\sqrt{A^2 + 4B(t+\tau)}$$

$$x_{ox} = \frac{1}{2}\left[\sqrt{\left(\frac{B}{B/A}\right)^2 + 4B(t+\tau)} - \frac{B}{B/A}\right]$$

$$\tau = 3{,}9\,s = 1{,}083 \cdot 10^{-3}\,h$$

$$B = C_1 e^{-\frac{E_1}{k_B T}} = 0{,}423\ \frac{\mu m}{h}$$

$$\frac{B}{A} = C_2 e^{-\frac{E_2}{k_B T}} = 1{,}392\ \frac{\mu m}{h}$$

$x_{ox} =$ (a) $0{,}023\ \mu m$ (b) $0{,}155\ \mu m$ (c) $0{,}516\ \mu m$

2 – Ternarne legure

Wednesday, May 4, 2022 9:33 AM

$$In_x Ga_{1-x} As = (InAs)_x (GaAs)_{1-x}$$

$$\underbrace{a_{InGaAs}}_{a_{InP}} = x a_{InAs} + (1-x) a_{GaAs}$$

$$a_{InP} = x a_{InAs} - x a_{GaAs} + a_{GaAs}$$

$$x = \frac{a_{InP} - a_{GaAs}}{a_{InP} - a_{GaAs}} = 0{,}5333$$

$$\chi_{InGaAs} = x \chi_{InAs} + (1-x) \chi_{GaAs}$$

$$\chi_{InGaAs} = 4{,}5233 \text{ eV}$$

$$In_x Al_{1-x} As = (InAs)_x (AlAs)_{1-x}$$

$$a_{InP} = x a_{InAs} + (1-x) a_{AlAs}$$

$$x = \frac{a_{InP} - a_{AlAs}}{a_{InAs} - a_{AlAs}} = 0{,}5251$$

$$\chi_{InAlAs} = x \chi_{InAs} + (1-x) \chi_{AlAs} = 4{,}2504 \text{ eV}$$

$$y = 1-x$$

$$E_g(Ga_y In_{1-y} As) = 0{,}7698 \text{ eV}$$

$$E_g(Al_y In_{1-y} As) = 1{,}5301 \text{ eV}$$

3 – Tip I i Tip III heterostrukture

Wednesday, May 4, 2022 9:47 AM

АНДЕРСОНОВО ПРАВИЛО → Вакуумски нивои два материјала који формирају хетероспој треба да буду поравнати

① GaAs ② AlAs

$$\Delta E_c = E_{c_2} - E_{c1} = (-\chi_2) - (-\chi_1) = \chi_1 - \chi_2$$

$$\Delta E_v = E_{v_2} - E_{v1} = (-\chi_2 - E_{g2}) - (-\chi_1 - E_{g1}) = \chi_1 - \chi_2 + E_{g1} - E_{g2} = \Delta E_c - \Delta E_{g12}$$

$\Delta E_c = 0{,}56\,eV$ $\Delta E_v = -0{,}17\,eV$

InAs – GaSb

$\Delta E_c = 0{,}86\,eV$ $\Delta E_v = 0{,}46\,eV$

4 – p+ AlGaAs I n- GaAs

Wednesday, May 4, 2022 10:06 AM

$$\overline{E}_c^A = E_c^A - \Delta E_c$$

$$\overline{E}_v^A = E_v^A + \Delta E_v$$

5 – Tip II n-p i p-n

Wednesday, May 4, 2022 10:28 AM

6 – Tip III heterospoja

Wednesday, May 4, 2022 10:35 AM

$$E_e = E_c^{InAs} + \varepsilon_e = E_c^{InAs} + \frac{\hbar^2\pi^2}{2m_e a^2}$$

$$E_h = E_v^{GaSb} - \varepsilon_h = E_v^{GaSb} - \frac{\hbar^2\pi^2}{2m_h a^2}$$

$$E_e = E_h$$

$$E_c^{InAs} + \varepsilon_e = E_v^{GaSb} - \varepsilon_h$$

$$E_v^{GaSb} - E_c^{InAs} = \varepsilon_e + \varepsilon_h = \frac{\hbar^2\pi^2}{2a^2}\left(\frac{1}{m_e} + \frac{1}{m_h}\right)$$

Андерсоново правило

$$E_v^{GaSb} = -\chi^{GaSb} - E_g^{GaSb}$$

$$E_c^{InAs} = -\chi^{InAs}$$

Kvantna jama 1

Wednesday, May 4, 2022 10:54 AM

$$-\frac{\hbar^2}{2}\nabla\left(\frac{1}{m(\vec{r})}\nabla\right)\chi(\vec{r}) + V(\vec{r})\chi(\vec{r}) = E\chi(\vec{r})$$

$$\hat{H}^+ = \hat{H} \qquad \hat{h} = \nabla\left(\frac{1}{m}\nabla\right) \qquad \hat{h}^+ = \hat{h}$$

$$\int_V \eta^* \hat{H}\chi\, dV = \int_V (\hat{H}\eta)^* \chi\, dV$$

$$\int_V \eta^* \nabla\left(\frac{1}{m}\nabla\chi\right) dV = \int\left(\nabla\left(\frac{1}{m}\nabla\eta\right)\right)^* \chi\, dV$$

$$\operatorname{div}\left(\eta^*\frac{1}{m}\operatorname{grad}\chi\right) = \operatorname{grad}\eta^*\frac{1}{m}\operatorname{grad}\chi + \eta^*\operatorname{div}\left(\frac{1}{m}\operatorname{grad}\chi\right)$$

$$\int_V \operatorname{div}\left(\eta^*\frac{1}{m}\operatorname{grad}\chi\right) dV = \oint_S \eta^*\frac{1}{m}\operatorname{grad}\chi\, d\vec{S} = 0$$

$$\int \operatorname{grad}\eta^*\frac{1}{m}\operatorname{grad}\chi\, dV = -\int \eta^*\operatorname{div}\left(\frac{1}{m}\operatorname{grad}\chi\right) dV \qquad (1)$$

$$\operatorname{div}\left(\operatorname{grad}\eta^* \frac{1}{m}\chi\right) = \operatorname{grad}\eta^* \frac{1}{m}\operatorname{grad}\chi + \chi \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\eta^*\right)$$

$$\int\limits_V \operatorname{div}\left(\operatorname{grad}\eta^* \frac{1}{m}\chi\right) = \oint\limits_S \operatorname{grad}\eta^* \frac{1}{m}\chi \, d\vec{S} = 0$$

$$\int\limits_V \operatorname{grad}\eta^* \frac{1}{m}\operatorname{grad}\chi \, dV = -\int\limits_V \chi \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\eta^*\right) \quad (2)$$

$$\int\limits_V \chi \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\eta^*\right) dV = \int\limits_V \eta^* \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\chi\right) dV \qquad (1) \text{ и } (2)$$

$$\int\limits_V \chi \left[\operatorname{div}\left(\frac{1}{m}\operatorname{grad}\eta\right)\right]^* dV = \int\limits_V \eta^* \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\chi\right) dV$$

$$\int\limits_V \eta^* \nabla\left(\frac{1}{m}\nabla\chi\right) dV = \int\limits_V \left(\nabla\left(\frac{1}{m}\nabla\eta\right)\right)^* \chi \, dV$$

MBE 1

Wednesday, April 27, 2022 9:03 AM

$$\boxed{F = \frac{1}{4} n \bar{c}} \qquad \bar{c} = v_{sr} \qquad \bar{c} = \sqrt{\frac{8}{3\pi}}\, c_{rms} \qquad c_{rms} = v_{eff} = \sqrt{\frac{3k_BT}{m}}$$

$$\boxed{p = n k_B T} \qquad \boxed{\bar{c} = \sqrt{\frac{8k_BT}{\pi m}}} \qquad \frac{1}{2} m c_{rms}^2 = \frac{3}{2} k_B T$$

$$F = \frac{1}{4} n \bar{c} = \frac{1}{4} \frac{p}{k_BT} \sqrt{\frac{8}{3\pi}} \sqrt{\frac{3k_BT}{m}} = \boxed{\frac{p}{\sqrt{2\pi m k_B T}}}$$

$$p = 5 \cdot 10^{-11}\ \text{mbar} = 5 \cdot 10^{-11} \cdot 10^{-3}\, 10^{5}\ \text{Pa} = 5 \cdot 10^{-9}\ \text{Pa}$$

$$m = \frac{M_{H_2}}{N_A} = \frac{2{,}01588 \cdot 10^{-3}\ \text{kg/mol}}{6{,}02214 \cdot 10^{23}\ \frac{1}{\text{mol}}} \qquad k_B = 1{,}381 \cdot 10^{-23}\ \frac{\text{J}}{\text{K}}$$

$$F_{bg} = 5{,}3563 \cdot 10^{14}\ \frac{1}{m^2 s} = 5{,}4 \cdot 10^{14}\ \frac{1}{m^2 s}$$

$$\overset{\text{GaAs}}{a} = 0{,}5653\ \text{nm} \rightarrow (0\ 0\ 1) \longrightarrow \bar{N} = \frac{2}{a^2}$$

$$t_{bg} = \frac{\bar{N}}{\xi F_{bg}} = \frac{2}{\xi a^2 F_{bg}} = 1{,}2 \cdot 10^{4}\ s \approx 3h$$

ВРЕМЕ ДА СЕ НАРАСТЕ МОНОСЛОЈ H_2

= ВРЕМЕ ТОКОМ КОТ ЈЕ ПОВРШИНА

ЧИСТА ОД П. ГАСА

MBE 2

Wednesday, April 27, 2022 9:41 AM

$$F_K = \frac{p_K}{\sqrt{2\pi m_{Ga} k_B T_K}}$$ ФЛУКС НА ОТВОРУ К-ЋЕЛИЈЕ

$$\frac{\left[\frac{1}{m^2}\right]}{\left[\frac{1}{m^2 s}\right]} \rightarrow [s]$$

$\pi r^2 F_K$ БРЗИНА КОЈОМ Ga НАПУШТАЈУ К-ЋЕЛИЈУ

$$\frac{\pi r^2 F_K}{2\pi R^2} = \frac{1}{2}\left(\frac{r}{R}\right)^2 F_K$$ ФЛУКС Ga НА ПОЛУСФЕРУ

$$F_{Ga} = 2 \cdot \frac{1}{2}\left(\frac{r}{R}\right)^2 F_K$$ ФЛУКС Ga НА ПЛОЧИЦУ

$$t_{ML} = \frac{\bar{N}}{\xi F_{Ga}} = 2\left(\frac{R}{ar}\right)^2 \frac{\sqrt{2\pi m_{Ga} k_B T_K}}{p_K} \approx 5s$$

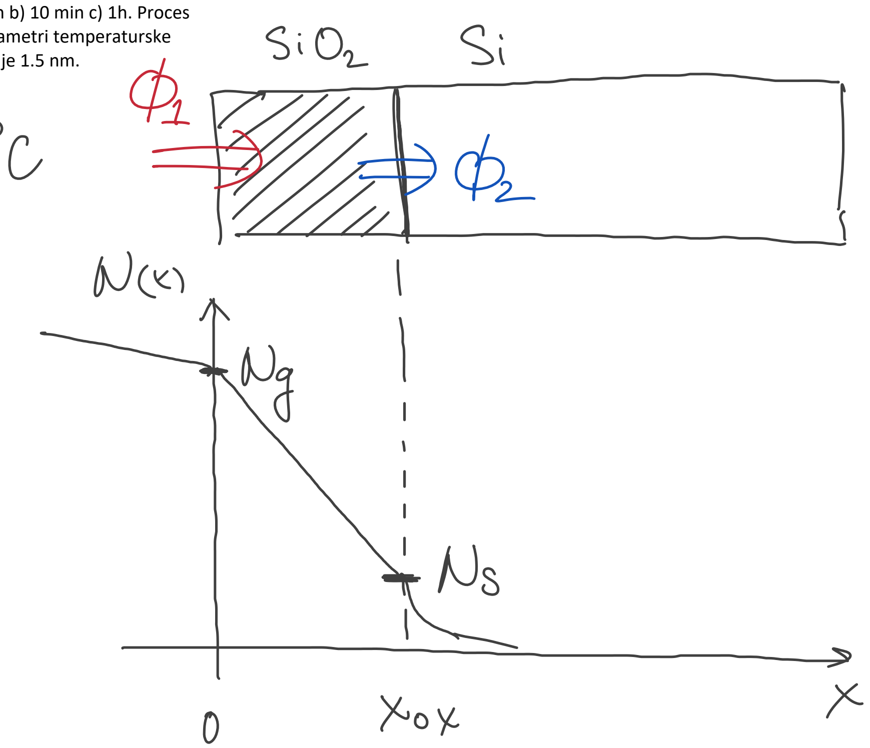
$$\frac{t_{ML}}{t_{bg}} = 5 \cdot 10^{-4} \gg 10^{-6}$$ $\xi_{H_2} = 1$

Oksidacija

Wednesday, March 16, 2022 10:43 AM

Deal – Grove

Izračunati debljinu oksidnog sloja, naraslog na silicijumskoj podlozi posle a) 1 min b) 10 min c) 1h. Proces oksidacije se odvija u vlažnoj atmosferi kiseonika na 1050 step. C. Poznati su parametri temperaturske zavisnosti Deal-Grove-ovih koeficijenata, dati u tabeli. Debljina prirodnog oksida je 1.5 nm.



$x_{ox} = ?$ (a) 1 min (b) 10 min (c) 1 h $t = 1050\,^\circ C$

$C e^{-\frac{E_a}{k_B T}}$

	B	B/A
C	$2{,}14 \cdot 10^{2}\ \frac{\mu m}{h}$	$8{,}95 \cdot 10^{7}\ \mu m / h$
E	$0{,}71\ eV$	$2{,}05\ eV$

$x_0 = 1{,}5\ nm$

$$\frac{\partial N}{\partial t} = \frac{\partial^2 N}{\partial x^2} = 0 \Rightarrow \frac{dN}{dx} = a \Rightarrow N(x) = ax + b$$

$$N(0) = b = N_g \qquad N(x_{ox}) = N_s = a x_{xo} + N_g$$

$$a = \frac{N_s - N_g}{x_{ox}} = -\frac{N_g - N_s}{x_{ox}}$$

$$N(x) = N_g - \frac{N_g - N_s}{x_{ox}} x$$

$$\Phi_1 = -D \frac{dN}{dx} = D \frac{N_g - N_s}{x_{ox}} \qquad \Phi_2 = k(N_s - N_e) \qquad k = k(T) \sim e^{-\frac{E}{k_B T}}$$

$$\Phi_1 = \Phi_2 = \Phi$$

$$k(N_s - N_e) = D \frac{N_g - N_s}{x_{ox}} \qquad Si + O_2 \rightarrow SiO_2$$

$$\boxed{v = \frac{dx_{ox}}{\ldots} = \frac{\Phi}{\ldots}} \qquad \Phi \Delta t A = N_{ox} \Delta V = N_{ox} A \Delta x$$

$$\boxed{v = \frac{dx}{dt} = \frac{\Phi}{N_{ox}}} \qquad \Phi \Delta t A = N_{ox} \Delta V_{ox} = N_{ox} A \Delta x_{ox}$$

$$\frac{\Delta x_{ox}}{\Delta t} = \frac{\Phi}{N_{ox}}$$

$$N_g - N_e = N_g - N_s + N_s - N_e = N_g - N_s + \frac{D}{k x_{ox}}(N_g - N_s)$$

$$N_g - N_e = (N_g - N_s)\left(1 + \frac{D}{k x_{ox}}\right)$$

$$N_g - N_s = \frac{N_g - N_e}{1 + \frac{D}{k x_{ox}}}$$

$$\Phi = \frac{D}{x_{ox}}(N_g - N_s) = \frac{D}{x_{ox}} \frac{N_g - N_e}{1 + \frac{D}{k x_{ox}}}$$

$$v = \frac{\Phi}{N_{ox}} = \frac{D}{N_{ox} x_{ox}} \frac{N_g - N_e}{1 + \frac{D}{k x_{ox}}}$$

$$\boxed{\frac{dx_{ox}}{dt} = \frac{D}{N_{ox}} \frac{N_g - N_e}{x_{ox} + \frac{D}{k}}}$$

$$\left(x_{ox} + \frac{D}{k}\right) dx_{ox} = \frac{D}{N_{ox}}(N_g - N_e)\, dt$$

$$\frac{x_{ox}^2}{2} + \frac{D}{k} x_{ox} = \frac{D(N_g - N_e)}{N_{ox}} t + C_1 \quad / \cdot 2$$

N_{ox}

$$x_{ox}^2 + \underbrace{\frac{2D}{k}}_{A} x_{ox} = \underbrace{\frac{2D(N_g - N_e)}{N_{ox}}}_{B} t + \underbrace{C}_{B\tau}$$

$$x_{ox}^2 + A x_{ox} = B(t+\tau) \qquad x_{ox}(0) = x_0 \qquad x_0^2 + A x_0 = B\tau \Rightarrow \tau = \frac{x_0^2 + A x_0}{B}$$

$$x_{ox}^2 + A x_{ox} - B(t+\tau) = 0$$

$$x_{ox} = -\frac{A}{2} \oplus \sqrt{\frac{A^2}{4} + B(t+\tau)} = -\frac{A}{2} + \frac{1}{2}\sqrt{A^2 + 4B(t+\tau)}$$

$$x_{ox} = \frac{1}{2}\left[\sqrt{\left(\frac{B}{B/A}\right)^2 + 4B(t+\tau)} - \frac{B}{B/A}\right]$$

$$\tau = 3{,}9\,s = 1{,}083 \cdot 10^{-3}\,h$$

$$B = C_1 e^{-\frac{E_1}{k_B T}} = 0{,}423\,\frac{\mu m}{h}$$

$$\frac{B}{A} = C_2 e^{-\frac{E_2}{k_B T}} = 1{,}392\,\frac{\mu m}{h}$$

$x_{ox} =$ (a) $0{,}023\,\mu m$ (b) $0{,}155\,\mu m$ (c) $0{,}516\,\mu m$

2 – Ternarne legure

Wednesday, May 4, 2022 9:33 AM

$$In_x Ga_{1-x} As = (InAs)_x (GaAs)_{1-x}$$

$$\underbrace{a_{InGaAs}}_{a_{InP}} = x a_{InAs} + (1-x) a_{GaAs}$$

$$a_{InP} = x a_{InAs} - x a_{GaAs} + a_{GaAs}$$

$$x = \frac{a_{InP} - a_{GaAs}}{a_{InP} - a_{GaAs}} = 0,5333$$

$$\chi_{InGaAs} = x \chi_{InAs} + (1-x) \chi_{GaAs}$$

$$\chi_{InGaAs} = 4,5233 eV$$

$$In_x Al_{1-x} As = (InAs)_x (AlAs)_{1-x}$$

$$a_{InP} = x a_{InAs} + (1-x) a_{AlAs}$$

$$x = \frac{a_{InP} - a_{AlAs}}{a_{InAs} - a_{AlAs}} = 0,5251$$

$$\chi_{InAlAs} = x \chi_{InAs} + (1-x) \chi_{AlAs} = 4,2504 eV$$

$$y = 1-x$$

$$E_g (Ga_y In_{1-y} As) = 0,7698 eV$$

$$E_g (Al_y In_{1-y} As) = 1,5301 eV$$

3 – Tip I i Tip III heterostrukture

Wednesday, May 4, 2022 9:47 AM

АНДЕРСОНОВО ПРАВИЛО → Вакуумски нивои два материјала који формирају хетероспој треба да буду поравнати

① GaAs ② AlAs

$$\Delta E_c = E_{c_2} - E_{c1} = (-\chi_2) - (-\chi_1) = \chi_1 - \chi_2$$

$$\Delta E_v = E_{v_2} - E_{v1} = (-\chi_2 - E_{g2}) - (-\chi_1 - E_{g1}) = \chi_1 - \chi_2 + E_{g1} - E_{g2} = \Delta E_c - \Delta E_{g12}$$

$\Delta E_c = 0{,}56\,eV$ $\Delta E_v = -0{,}17\,eV$

InAs – GaSb

$\Delta E_c = 0{,}86\,eV$ $\Delta E_v = 0{,}46\,eV$

4 – p+ AlGaAs I n- GaAs

Wednesday, May 4, 2022 10:06 AM

$$\overline{E}_c^A = E_c^A - \Delta E_c$$

$$\overline{E}_v^A = E_v^A + \Delta E_v$$

5 – Tip II n-p i p-n

Wednesday, May 4, 2022 10:28 AM

6 – Tip III heterospoja

Wednesday, May 4, 2022 10:35 AM

$$E_e = E_c^{InAs} + \varepsilon_e = E_c^{InAs} + \frac{\hbar^2\pi^2}{2m_e a^2}$$

$$E_h = E_v^{GaSb} - \varepsilon_h = E_v^{GaSb} - \frac{\hbar^2\pi^2}{2m_h a^2}$$

$$E_e = E_h$$

$$E_c^{InAs} + \varepsilon_e = E_v^{GaSb} - \varepsilon_h$$

$$E_v^{GaSb} - E_c^{InAs} = \varepsilon_e + \varepsilon_h = \frac{\hbar^2\pi^2}{2a^2}\left(\frac{1}{m_e} + \frac{1}{m_h}\right)$$

Андерсоново правило

$$E_v^{GaSb} = -\chi^{GaSb} - E_g^{GaSb}$$

$$E_c^{InAs} = -\chi^{InAs}$$

Kvantna jama 1

Wednesday, May 4, 2022 10:54 AM

$$-\frac{\hbar^2}{2}\nabla\left(\frac{1}{m(\vec{r})}\nabla\right)\chi(\vec{r}) + V(\vec{r})\chi(\vec{r}) = E\chi(\vec{r})$$

$$\hat{H}^+ = \hat{H} \qquad \hat{h} = \nabla\left(\frac{1}{m}\nabla\right) \qquad \hat{h}^+ = \hat{h}$$

$$\int_V \eta^* \hat{H}\chi \, dV = \int_V (\hat{H}\eta)^* \chi \, dV$$

$$\int_V \eta^* \nabla\left(\frac{1}{m}\nabla\chi\right) dV = \int \left(\nabla\left(\frac{1}{m}\nabla\eta\right)\right)^* \chi \, dV$$

$$\operatorname{div}\left(\eta^* \frac{1}{m}\operatorname{grad}\chi\right) = \operatorname{grad}\eta^* \frac{1}{m}\operatorname{grad}\chi + \eta^* \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\chi\right)$$

$$\int_V \operatorname{div}\left(\eta^* \frac{1}{m}\operatorname{grad}\chi\right) dV = \oint_S \eta^* \frac{1}{m}\operatorname{grad}\chi \, d\vec{S} = 0$$

$$\int \operatorname{grad}\eta^* \frac{1}{m}\operatorname{grad}\chi \, dV = -\int \eta^* \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\chi\right) dV \quad (1)$$

$$\operatorname{div}\left(\operatorname{grad}\eta^* \frac{1}{m}\chi\right) = \operatorname{grad}\eta^* \frac{1}{m}\operatorname{grad}\chi + \chi \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\eta^*\right)$$

$$\int_V \operatorname{div}\left(\operatorname{grad}\eta^* \frac{1}{m}\chi\right) = \oint_S \operatorname{grad}\eta^* \frac{1}{m}\chi \, d\vec{S} = 0$$

$$\int_V \operatorname{grad}\eta^* \frac{1}{m}\operatorname{grad}\chi \, dV = -\int_V \chi \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\eta^*\right) \quad (2)$$

$$\int_V \chi \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\eta^*\right) dV = \int_V \eta^* \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\chi\right) dV \qquad (1) \text{ и } (2)$$

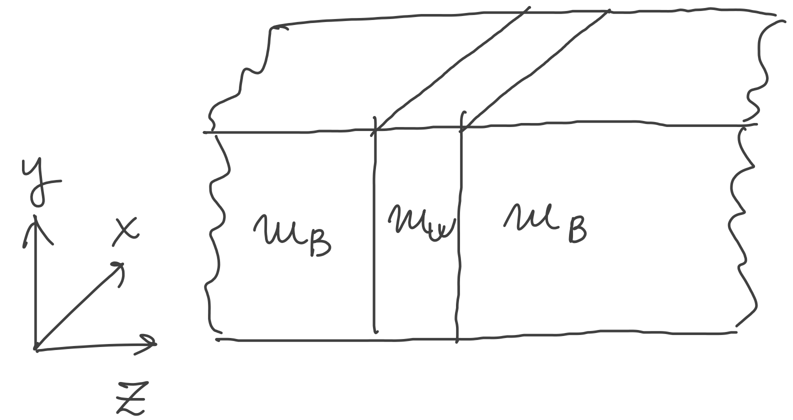
$$\int_V \chi \left[\operatorname{div}\left(\frac{1}{m}\operatorname{grad}\eta\right)\right]^* dV = \int_V \eta^* \operatorname{div}\left(\frac{1}{m}\operatorname{grad}\chi\right) dV$$

$$\int_V \eta^* \nabla\left(\frac{1}{m}\nabla\chi\right) dV = \int_V \left(\nabla\left(\frac{1}{m}\nabla\eta\right)\right)^* \chi \, dV$$

Jednačina

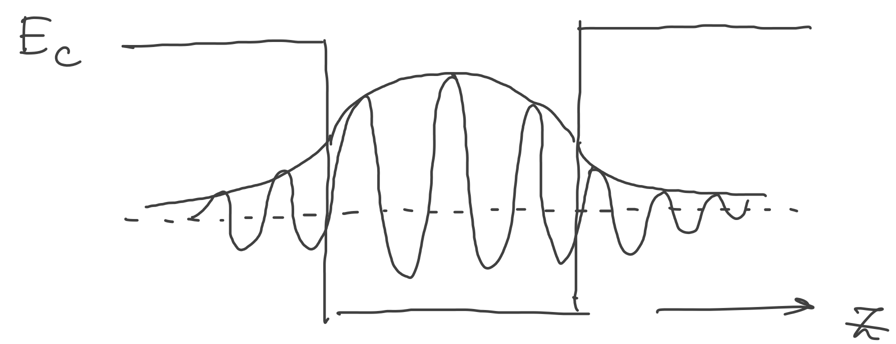
Tuesday, May 10, 2022 4:16 PM

$$-\frac{\hbar^2}{2}\nabla\left(\frac{1}{m(\vec{r})}\nabla\right)\chi(\vec{r}) + V(\vec{r})\chi(\vec{r}) = E\chi(\vec{r})$$



$$m(\vec{r}) = m(z)$$
$$V(\vec{r}) = V(z)$$

$$\chi(\vec{r}) = C e^{ik_x x} e^{ik_y y}\psi(z)$$



$$|\chi|^2 = |\psi(z)|^2$$

$$\chi(\vec{r}) = C e^{i\vec{k}_\parallel \vec{r}_\parallel}\psi(z)$$

$$\Psi(z) = u_{n0}(z)\chi(z)$$

$$\vec{k}_\parallel = \vec{k}_t = k_x\vec{e}_x + k_y\vec{e}_y$$

$$-\frac{\hbar^2}{2m}\nabla\left(\frac{1}{m(z)}\right)\nabla\left[e^{ik_x x}e^{ik_y y}\psi(z)\right] + V(z)e^{i\vec{k}_\parallel\vec{r}_\parallel}\psi = E e^{i\vec{k}_\parallel\vec{r}_\parallel}\psi$$

$$\left(\frac{\partial}{\partial x}\vec{e}_x + \frac{\partial}{\partial y}\vec{e}_y + \frac{\partial}{\partial z}\vec{e}_z\right)\left(e^{ik_x x}e^{ik_y y}\psi(z)\right) = ik_x e^{ik_x x}e^{ik_y y}\psi\,\vec{e}_x + ik_y e^{ik_x x}e^{ik_y y}\psi\,\vec{e}_y$$
$$+\ e^{ik_x x}e^{ik_y y}\frac{\partial\psi}{\partial z}\vec{e}_z$$

$$\nabla\left[\frac{1}{m(z)}\left(\qquad\qquad\right)\right]$$

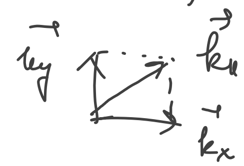
$$= \frac{\partial}{\partial}\left[\frac{1}{\ }ik_x\ e^{ik_x x}e^{ik_y y}\psi(z)\right] + \frac{\partial}{\partial}\left[\frac{1}{m(z)}ik_y\ e^{ik_x x}e^{ik_y y}\psi(z)\right]$$

$$\frac{\partial}{\partial x}\left[\frac{1}{m(z)} e^{ik_x x} e^{ik_y y} \psi(z)\right] + \frac{\partial}{\partial y}\left[\frac{1}{m(z)} e^{ik_x x} e^{ik_y y} \psi(z)\right]$$

$$+ \frac{\partial}{\partial z}\left[\frac{1}{m(z)} e^{ik_x x} e^{ik_y y} \frac{\partial \psi}{\partial z}\right]$$

$$= \left[\frac{1}{m(z)}(-k_x^2)\psi(z) + \frac{1}{m(z)}(-k_y^2)\psi(z) + \frac{d}{dz}\left(\frac{1}{m(z)}\frac{d\psi}{dz}\right)\right] e^{i\vec{k}_\parallel \vec{r}_\parallel}$$

$$-\frac{\hbar^2}{2}\frac{d}{dz}\left(\frac{1}{m(z)}\frac{d\psi}{dz}\right) e^{i\vec{k}_\parallel \vec{r}_\parallel} + \left(\frac{\hbar^2 k_x^2}{2m(z)} + \frac{\hbar^2 k_y^2}{2m(z)}\right) e^{i\vec{k}_\parallel \vec{r}_\parallel}\psi(z) + V(z)\, e^{i\vec{k}_\parallel \vec{r}_\parallel}\psi(z)$$



$$= E\, e^{i\vec{k}_\parallel \vec{r}_\parallel}\psi(z)$$

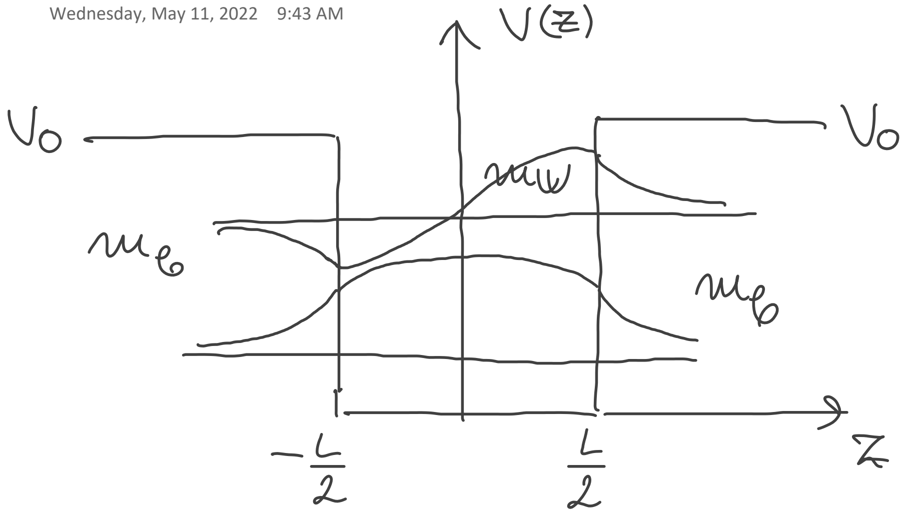
$$-\frac{\hbar^2}{2}\frac{d}{dz}\left(\frac{1}{m(z)}\frac{d\psi}{dz}\right) + \left(V(z) + \frac{\hbar^2 k_\parallel^2}{2m(z)}\right)\psi(z) = E\psi(z)$$

$$V_{eff}(z) = V(z) + \frac{\hbar^2 k_\parallel^2}{2m(z)} \qquad V(z) = V_h(z) - e\phi(z)$$

$\phi(z) = 0$ апр. равних зона $\left(N_D < 10^{15}\ cm^{-3}\right)$

Disperzione relacije

Wednesday, May 11, 2022 9:43 AM



$$V(z) = \begin{cases} V_0, & |z| \geq \frac{L}{2} \\ 0, & |z| < \frac{L}{2} \end{cases} \qquad m(z) = \begin{cases} m_b, & |z| \geq \frac{L}{2} \\ m_w, & |z| < \frac{L}{2} \end{cases}$$

$$\left[-\frac{\hbar^2}{2}\frac{d}{dz}\frac{1}{m(z)}\frac{d}{dz} + V(z)\right]\phi(z) = E\phi(z)$$

$$\phi\left(\frac{L^+}{2}\right) = \phi\left(\frac{L^-}{2}\right)$$

$$\frac{1}{m_b}\frac{d}{dz}\phi\left(\frac{L^+}{2}\right) = \frac{1}{m_w}\frac{d}{dz}\phi\left(\frac{L^-}{2}\right)$$

$$\phi(z) = \begin{cases} C_1 e^{-\kappa\left(|z| - \frac{L}{2}\right)}, & |z| \geq \frac{L}{2} \\ C_2 \cos(kz), & |z| < \frac{L}{2} \end{cases}$$

$$\kappa = \sqrt{\frac{2m_b(V_0 - E)}{\hbar^2}}$$

$$k = \sqrt{\frac{2m_w E}{\hbar^2}}$$

$$\left.\begin{aligned} C_1 &= C_2\cos\left(k\frac{L}{2}\right) \\ \frac{\kappa}{m_b}C_1 &= \frac{k}{m_w}C_2\sin\left(k\frac{L}{2}\right) \end{aligned}\right\} \quad \boxed{\kappa = \frac{m_b}{m_w}k\tan\left(\frac{kL}{2}\right)}$$

$$\phi(z) = \begin{cases} C_1 e^{-\kappa\left(z - \frac{L}{2}\right)}, & z > \frac{L}{2} \\ C_2 \sin(kz) & |z| \le \frac{L}{2} \\ -C_1 e^{\kappa\left(z + \frac{L}{2}\right)} & z < -\frac{L}{2} \end{cases}$$

$$C_1 = C_2 \sin k\frac{L}{2}$$

$$-\frac{\kappa C_1}{m_b} = \frac{k}{m_w} C_2 \cos\left(\frac{kL}{2}\right)$$

$$\boxed{\kappa = -\frac{m_b k}{m_w} \cot\left(\frac{kL}{2}\right)}$$

$$\left(k\frac{L}{2}\right)^2 = \frac{2m_w E}{\hbar^2}\left(\frac{L}{2}\right)^2$$

$$\left(\kappa\frac{L}{2}\right)^2 = \frac{2m_b V_0}{\hbar^2}\left(\frac{L}{2}\right)^2 - \frac{2m_b E}{\hbar^2}\left(\frac{L}{2}\right)^2 \quad \Big/ \frac{m_w}{m_b}$$

$$+\ \frac{m_w}{m_b}\left(\kappa\frac{L}{2}\right)^2 = \frac{2m_w V_0}{\hbar^2}\left(\frac{L}{2}\right)^2 - \frac{2m_w E}{\hbar^2}\left(\frac{L}{2}\right)^2$$

$$\left(k\frac{L}{2}\right)^2 + \frac{m_w}{m_b}\left(\kappa\frac{L}{2}\right)^2 = \frac{2m_w V_0}{\hbar^2}\left(\frac{L}{2}\right)^2$$

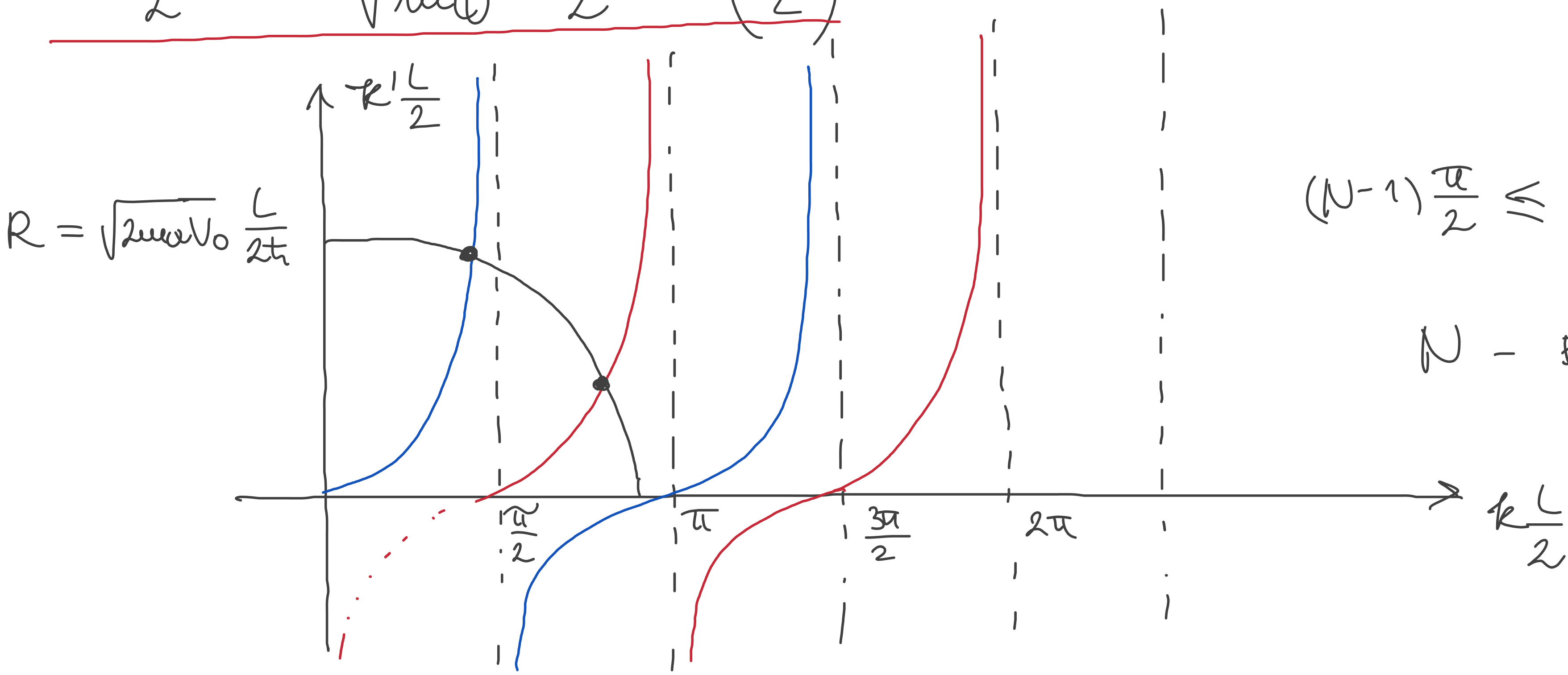
$$\kappa\frac{L}{2} = \frac{m_b}{m_w} k\frac{L}{2} \tan\left(\frac{kL}{2}\right) \quad \text{ПАРНА РЕШЕЊА}$$

$$\kappa\frac{L}{2} = -\frac{m_b}{m_w} k\frac{L}{2} \cot\left(\frac{kL}{2}\right) \quad \text{НЕПАРНА РЕШЕЊА}$$

$$\kappa' \equiv \kappa\sqrt{\frac{m_w}{m_b}}$$

$$k'\frac{L}{2} = \sqrt{\frac{m_b}{m_w}}\, k\frac{L}{2} \tan\left(\frac{kL}{2}\right) \qquad \left(k\frac{L}{2}\right)^2 + \left(k'\frac{L}{2}\right)^2 = R^2$$

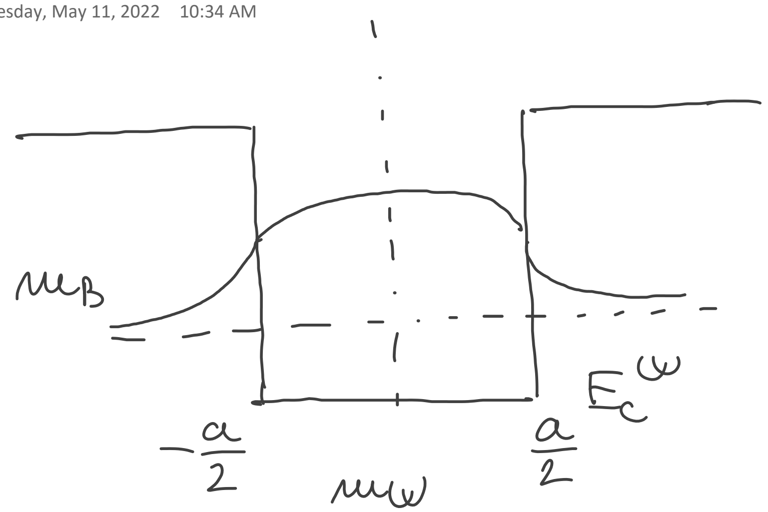
$$k'\frac{L}{2} = -\sqrt{\frac{m_b}{m_w}}\, k\frac{L}{2} \cot\left(\frac{kL}{2}\right) \qquad R = \sqrt{2m_w V_0}\,\frac{L}{2\hbar}$$



$$(N-1)\frac{\pi}{2} \leq \sqrt{2m_w V_0}\,\frac{L}{2\hbar} \leq N\frac{\pi}{2}$$

N – БРОЈ РЕШЕЊА

Wednesday, May 11, 2022 10:34 AM



$$V_0 = E_c^B - E_c^W$$

$$\Psi(z) = C \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} kz \qquad k = \sqrt{\frac{2m_W (E - E_c^W)}{\hbar^2}}$$

$$\Psi(z) = D e^{\pm \kappa z} \qquad \kappa = \sqrt{\frac{2m_B (E_c^B - E)}{\hbar^2}}$$

$$\Psi\left(\frac{a}{2}\right) = C \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} \left(\frac{ka}{2}\right) = D e^{-\kappa \frac{a}{2}} \quad (1)$$

$$\frac{1}{m_W} \frac{d\Psi}{dz}\Bigg|_{z = \frac{a}{2}^-} = \frac{1}{m_B} \frac{d\Psi}{dz}\Bigg|_{z = \frac{a}{2}^+}$$

$$\frac{1}{m_W} C k \begin{Bmatrix} -\sin \\ \cos \end{Bmatrix} \left(\frac{ka}{2}\right) = -\frac{D\kappa}{m_B} e^{-\frac{\kappa a}{2}} \quad (2)$$

$$\frac{(2)}{(1)} \qquad \boxed{\begin{Bmatrix} \tan \\ -\cot \end{Bmatrix} \left(\frac{ka}{2}\right) = \frac{m_W}{m_B} \frac{\kappa}{k}}$$

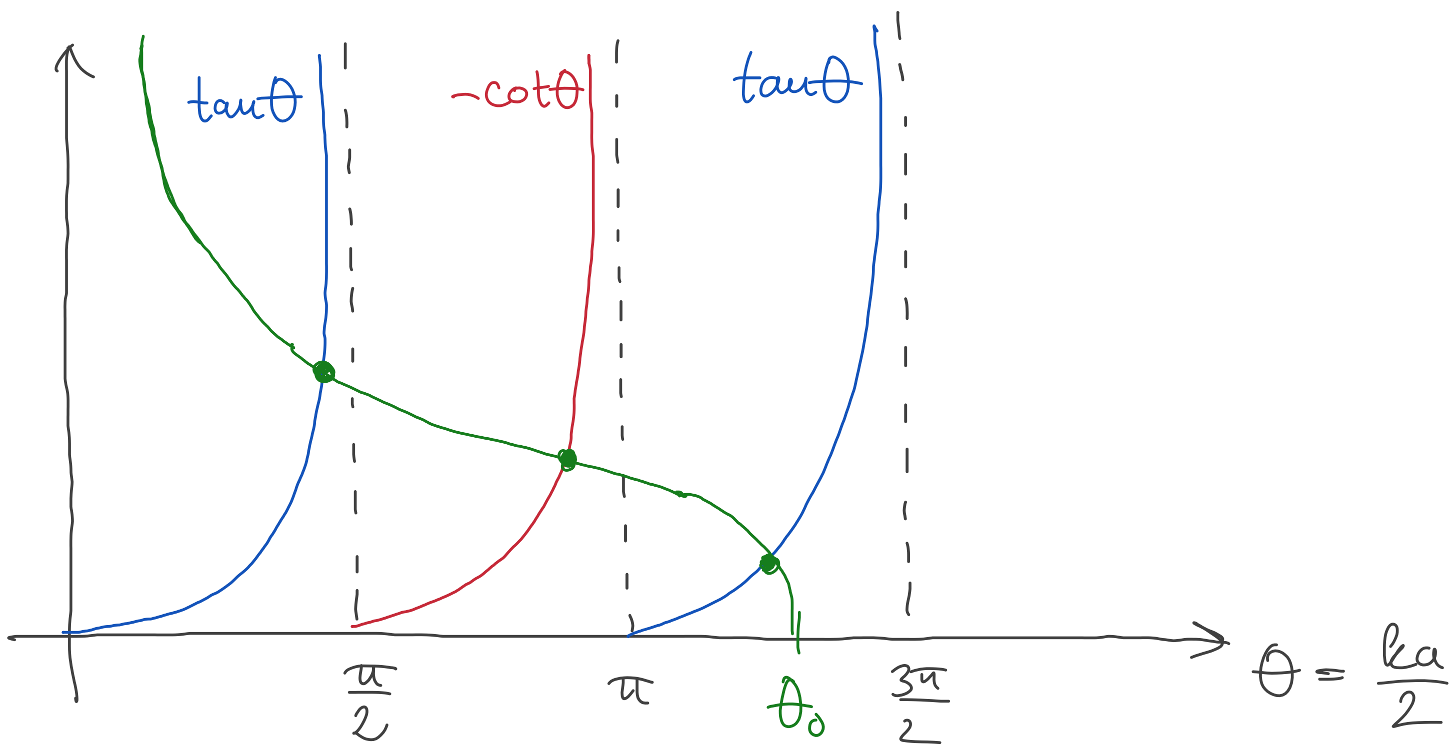
$$\frac{m_W}{m_B} \frac{\kappa}{k} = \sqrt{\frac{m_W^2}{m_B^2} \frac{2m_B (E_c^B - E)}{\hbar^2 k^2}} = \sqrt{\frac{m_W^2}{m_B^2} \frac{2m_B (E_c^B - E_c^W + E_c^W - E)}{\hbar^2 k^2}}$$

$$= \sqrt{\frac{m_W^2}{m_B} \frac{2V_0}{\hbar^2 k^2} - \frac{2m_W^2}{m_B} \frac{E - E_c^W}{\hbar^2 k^2}} = \sqrt{\frac{m_W}{m_B} \left(\frac{2m_W V_0}{\hbar^2 k^2} \left(\frac{2}{a}\right)^2 \left(\frac{a}{2}\right)^2 - 1 \right)}$$

$$\frac{\frac{2m_w}{\hbar^2}\left(E-E_c^{(w)}\right)}{k^2} = 1$$

$$\theta = \frac{ka}{2} \qquad \theta_0^2 = \frac{m_w V_0 a^2}{2\hbar^2} \qquad \frac{m_w}{m_B}\frac{\kappa}{k} = \sqrt{\frac{m_w}{m_B}\left(\frac{\theta_0^2}{\theta^2} - 1\right)}$$

$$\left\{\begin{matrix} \tan \\ -\cot \end{matrix}\right\} \theta = \sqrt{\frac{m_w}{m_B}\left(\frac{\theta_0^2}{\theta^2} - 1\right)}$$

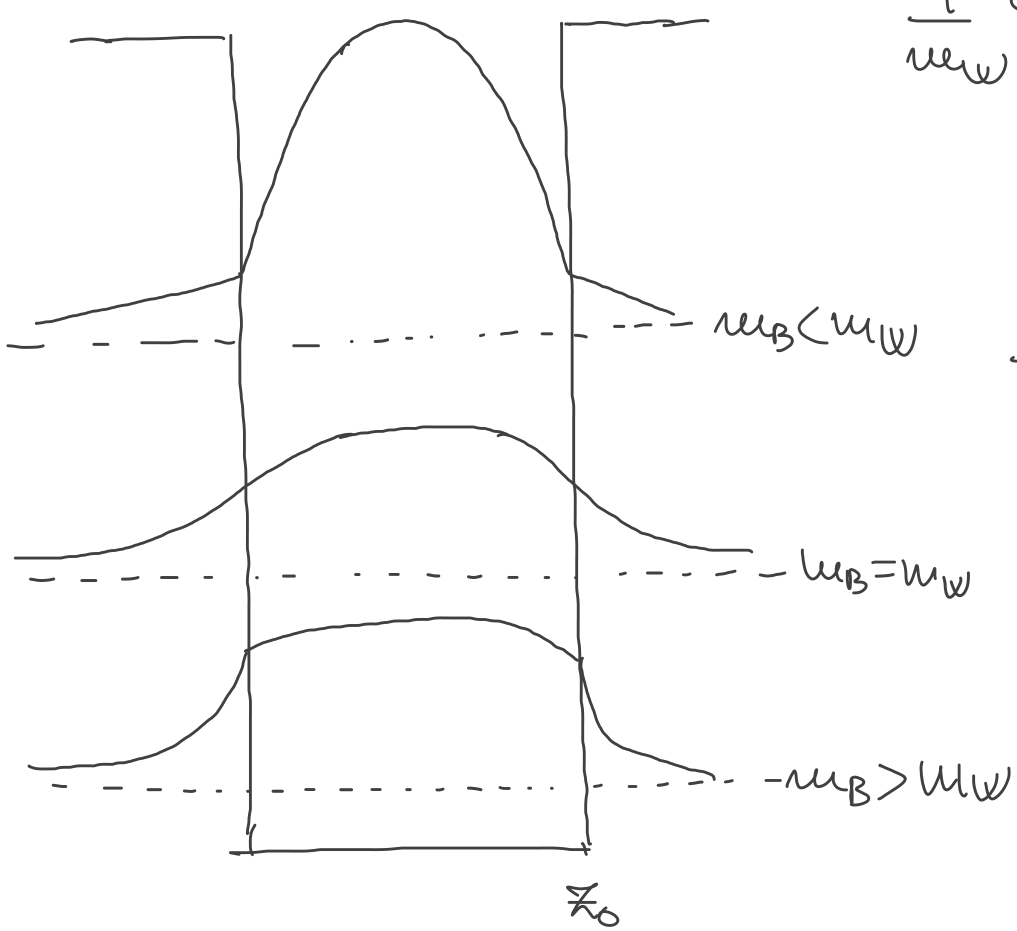


Granični uslovi

Wednesday, May 11, 2022 10:52 AM

Elektroni se nalaze u jami u kojoj se njihova efektivna masa značajno razlikuje od efektivne mase u barijeri. Kako izgledaju granični uslovi, razmotriti oba slučaja.

$m_W << m_B$ $\quad$ $m_W >> m_B$



$$\phi(z_0^-) = \phi_0(z_0^+)$$

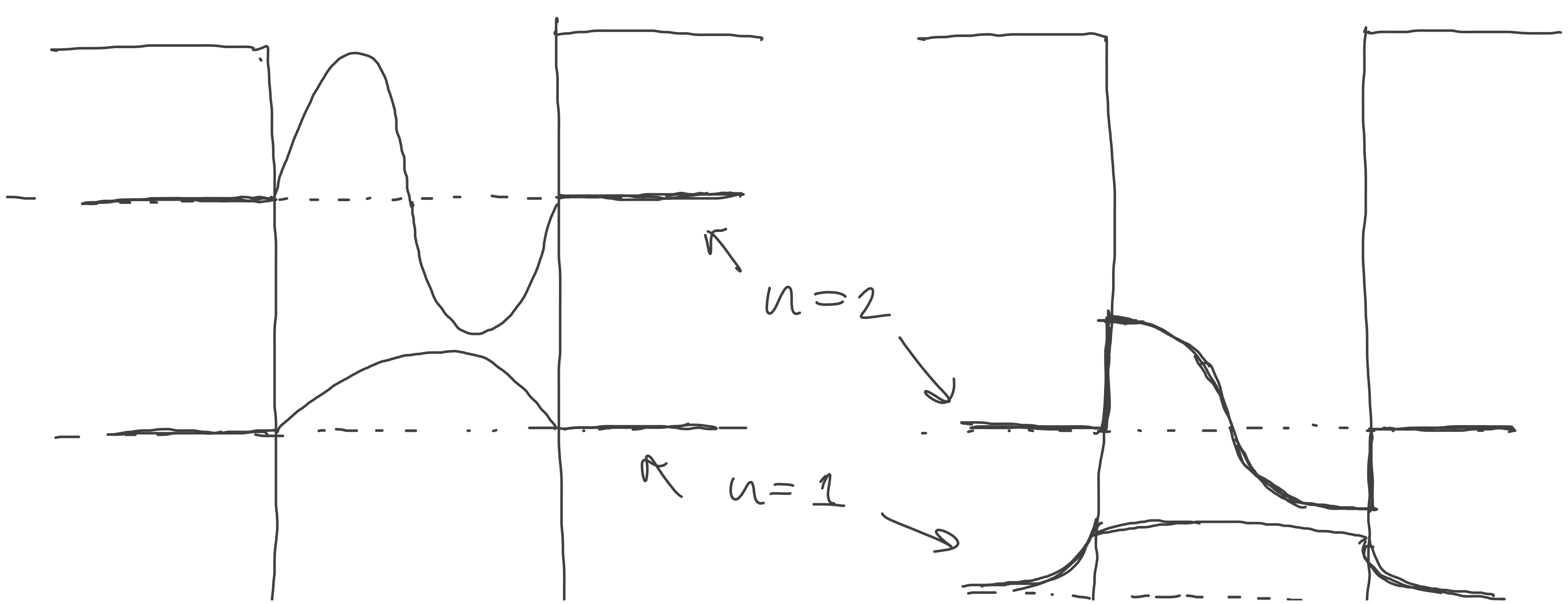
$$\frac{1}{m_W}\frac{d\phi}{dz}\bigg|_{z_0^-} = \frac{1}{m_B}\frac{d\phi}{dz}\bigg|_{z_0^+}$$

$m_B < m_W$ $\qquad$ $m_B > m_W$

$$\frac{1}{m_B} > \frac{1}{m_W} \qquad \frac{1}{m_B} < \frac{1}{m_W}$$

$$\frac{d\phi}{dz}\bigg|_W > \frac{d\phi}{dz}\bigg|_B \qquad \frac{d\phi}{dz}\bigg|_W < \frac{d\phi}{dz}\bigg|_B$$

$m_B << m_W$ $\qquad$ $m_B >> m_W$



$$\left.\frac{d\phi}{dz}\right|_w \gg \left.\frac{d\phi}{dz}\right|_B \qquad \left.\frac{d\phi}{dz}\right|_w \ll \left.\frac{d\phi}{dz}\right|_B$$

$$\phi_2(z) \sim \sin\left(\frac{2\pi z}{a}\right) \qquad \phi_2(z) \sim \sin\left(\frac{\pi z}{a}\right)$$

$$E_2 = \frac{\hbar^2}{2m_w}\frac{4\pi^2}{a^2} \qquad E_2 = \frac{\hbar^2}{2m_w}\frac{\pi^2}{a^2}$$

$$E_1 \to 0 \qquad m_B \to \infty$$

$$\begin{Bmatrix} \tan \\ -\cot \end{Bmatrix}\theta = \sqrt{\frac{m_w}{m_w}\left(\frac{\theta_0^2}{\theta^2} - 1\right)} \qquad \theta = \frac{ka}{2} \qquad \theta_0^2 = \frac{m_w V_0 a^2}{2\hbar^2}$$

$$k = \sqrt{\frac{2m_w E}{\hbar^2}} \qquad \left(\frac{2\theta}{a}\right)^2 = \frac{2m_w E}{\hbar^2} \qquad E = \frac{2\hbar^2\theta^2}{m_w a^2} \qquad V_0 = \frac{2\hbar^2\theta_0^2}{m_w a^2}$$

$$\theta \ll 1 \qquad \theta \ll \theta_0$$

$$\theta \approx \sqrt{\frac{m_w}{m_B}}\,\frac{\theta_0}{\theta} \qquad \frac{m_w a^2 E_1}{2\hbar^2} = \frac{m_w}{\sqrt{m_B}}\,\frac{a}{\hbar\sqrt{2}}\sqrt{V_0}$$

$$\theta^2 = \sqrt{\frac{m_w}{m_B}}\sqrt{\frac{m_w a^2 V_0}{2\hbar^2}} \qquad E_1 = \sqrt{\frac{2\hbar^2 V_0}{m_B a^2}}$$

$m_B \to \infty$ $\quad E_1 \to 0$

$(m_B >> m_W)$

Gustina stanja

Wednesday, May 18, 2022 9:16 AM

Odrediti gustinu stanja za beskonačno duboku kvantnu jamu širine a. Uporediti rezultat sa gustinom stanja za slobodne, 3D, elektrone.
Kolika je maksimalna koncentracija elektrona koju sistem može da ima, ako je samo prva podzona popunjena?

ρ_{2D} ... a

$$\rho_{3D}(E) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E} \qquad \left[\frac{1}{eV\,nm^3}\right]$$

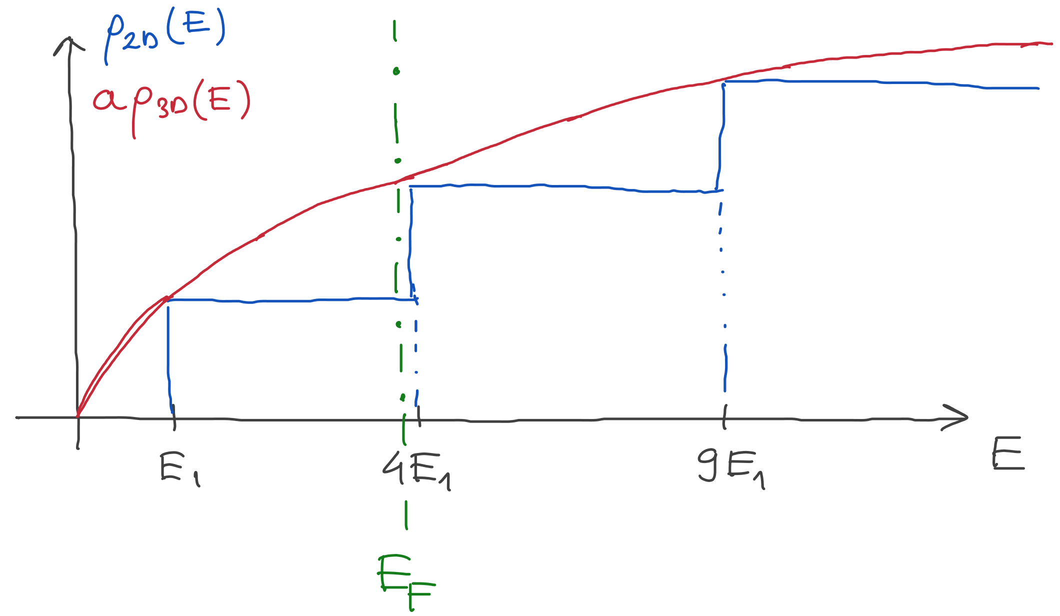
$$\rho_{3D}(E) = \frac{m}{\pi^2\hbar^3}\sqrt{2mE}$$

$$a\rho_{3D}(E) = \frac{ma}{\pi^2\hbar^3}\sqrt{2mE}$$

$$\rho_{2D}(E) = \frac{m}{\pi\hbar^2}\sum_{n=1}^{+\infty}\theta(E-\varepsilon_n) \qquad \varepsilon_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2$$

$$a\rho_{3D}(\varepsilon_n) = \frac{ma}{\pi^2\hbar^3}\sqrt{2m\varepsilon_n} = \frac{ma}{\pi^2\hbar^3}\sqrt{2m\frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2}$$

$$a\rho_{3D}(\varepsilon_n) = \frac{ma}{\pi^2\hbar^3}\,\frac{\hbar n\pi}{a} = \frac{mn}{\pi\hbar^2} = \rho_{2D}(\varepsilon_n)$$



$$n_{2D} = \frac{m}{\pi\hbar^2}\sum_j (E_F-\varepsilon_j)\,\theta(E_F-\varepsilon_j)$$

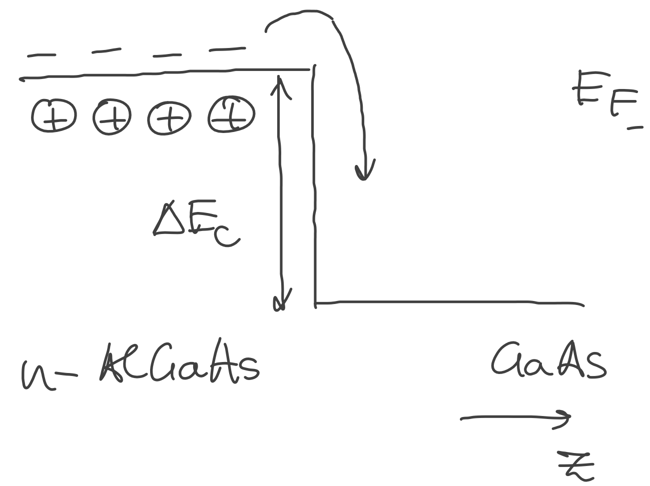
$$E_F \approx \varepsilon_2$$

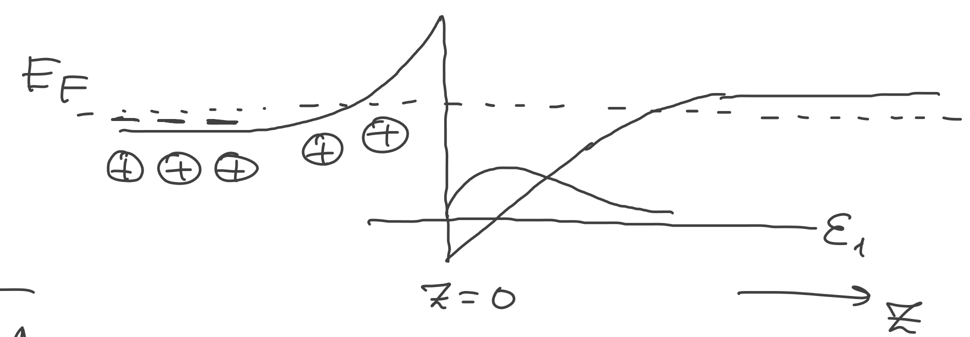
$$n_{2D}^{(max)} = \frac{m}{\pi\hbar^2}(\varepsilon_2-\varepsilon_1) = \frac{m}{\pi\hbar^2}(4-1)\frac{\hbar^2\pi^2}{2ma^2} = \frac{3\pi}{2a^2}$$

Jednačina i rešenja

Wednesday, May 18, 2022 9:30 AM

МОДУЛАЦИОНО ДОПИРАЊЕ





$$E_c(z) = E_c^0(z) - e\phi(z)$$

$$V(z) = \begin{cases} +\infty, & z<0 \\ eFz, & z>0 \end{cases}$$

$$\vec{F} = -\frac{d\phi}{dz}\vec{e}_z = F\vec{e}_z$$

$$\phi(z) = -Fz, \quad \phi(0) = 0$$

$$V(z) = (-e)(-Fz) = eFz$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dz^2} + eFz\right]\psi(z) = \varepsilon\psi(z)$$

$$z_0 = \left(\frac{\hbar^2}{2meF}\right)^{\frac{1}{3}} \qquad \varepsilon_0 = \left[\frac{(eF\hbar)^2}{2m}\right]^{\frac{1}{3}} = eFz_0$$

$$\bar{z} = \frac{z}{z_0} \qquad \bar{\varepsilon} = \frac{\varepsilon}{\varepsilon_0}$$

$$\frac{d^2\psi}{dz^2} = \frac{2meF}{\hbar^2}z\psi - \frac{2m\varepsilon}{\hbar^2}\psi \qquad \frac{d^2}{dz^2} = \frac{1}{z_0^2}\frac{d^2}{d\bar{z}^2}$$

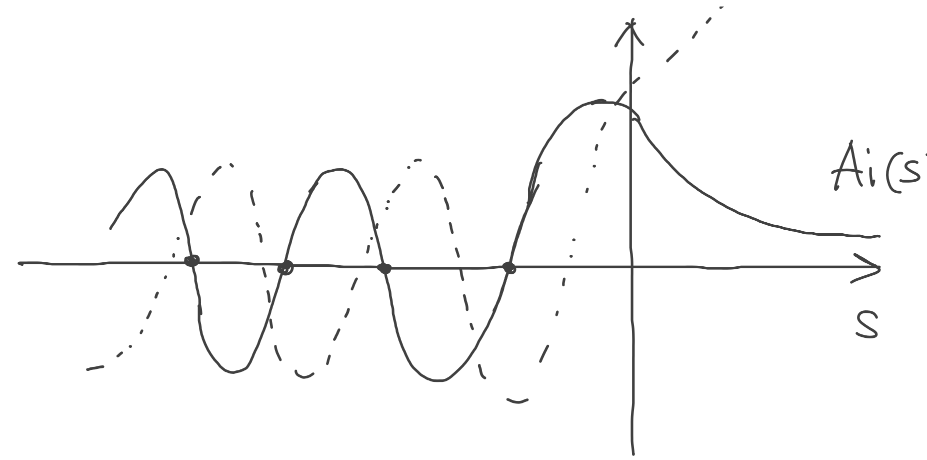
$$\frac{2m}{\hbar^2}\frac{(\hbar^2)^{\frac{2}{3}}}{(2m)^{\frac{2}{3}}(eF)^{\frac{2}{3}}}$$

$$\frac{d^2\psi}{d\bar{z}^2} = z_0^2\frac{1}{z_0^3}z\psi - \frac{2m\varepsilon}{\hbar^2}z_0^2\psi \qquad \frac{(2m)^{\frac{1}{3}}}{(\hbar^2)^{\frac{1}{3}}}\frac{1}{(eF)^{\frac{2}{3}}} = \frac{1}{\varepsilon_0}$$

$, Bi(s)$

$$\frac{d^2\Psi}{d\bar{z}^2} = \bar{z}\Psi - \bar{\varepsilon}\Psi \qquad s = \bar{z} - \bar{\varepsilon}$$

$$\frac{d^2\Psi}{ds^2} = s\Psi$$ Стоксова или Ејријева ј-на



$$\Psi(s) = C_1 Ai(s) + C_2 Bi(s)$$

$$s \to +\infty \Leftrightarrow \bar{z} \to +\infty \Rightarrow Bi(s) \to +\infty$$

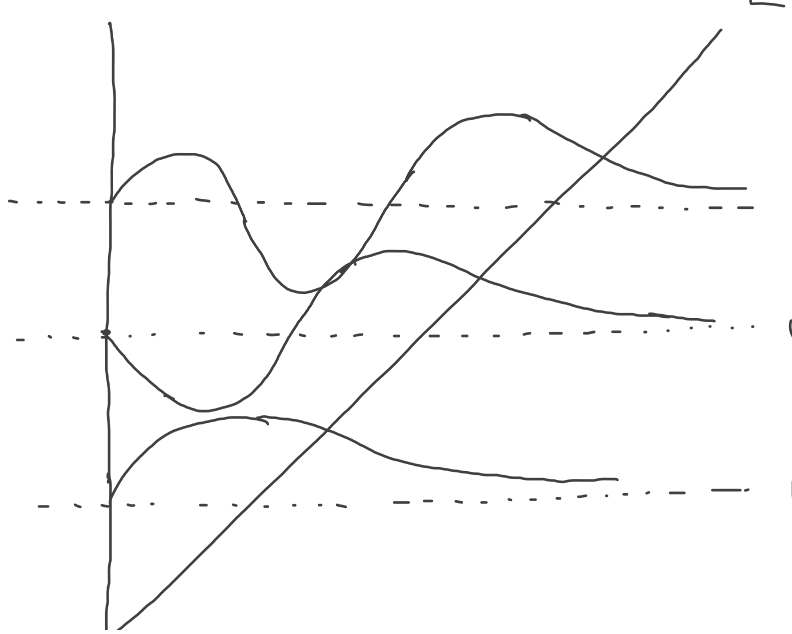
$$\Psi(z) = Ai(s) = Ai(\bar{z} - \bar{\varepsilon}) = Ai\left(\frac{z}{z_0} - \frac{\varepsilon}{\varepsilon_0}\right) = Ai\left(\frac{\frac{z\varepsilon_0}{z_0} - \varepsilon}{\varepsilon_0}\right) = Ai\left(\frac{\frac{zeFz_0}{z_0} - \varepsilon}{\varepsilon_0}\right)$$

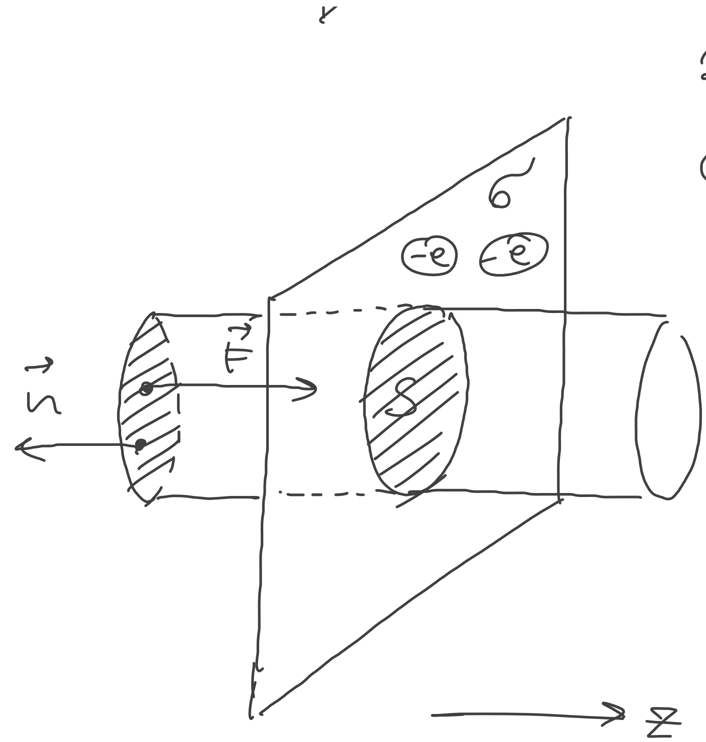
$$\Psi(z) = Ai\left(\frac{zeF - \varepsilon}{\varepsilon_0}\right)$$

$$\Psi(z=0) = 0 \qquad \Psi(\bar{z}=0) = \Psi(s = -\bar{\varepsilon}) = 0$$

$Ai(s) = 0 \qquad s = a_n = -c_n$ НУЛЕ ФУНКЦИЈЕ

$$\bar{\varepsilon}_n = c_n \qquad \varepsilon_n = c_n\varepsilon_0 = c_n\left[\frac{(eF\hbar)^2}{2m}\right]^{\frac{1}{3}}, \quad n = 1, 2, 3 \ldots$$





2DEG $\rightarrow n_{2D}$

$\sigma = -e n_{2D}$

$$\oint_S \vec{F} \cdot d\vec{S}$$

$$\frac{-e n_{2D}}{\varepsilon_0 \varepsilon_s} S = -F S$$

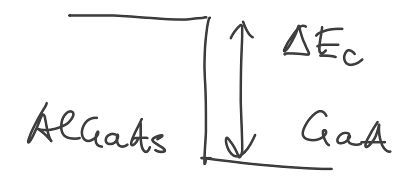
$$F = \frac{e n_{2D}}{\varepsilon_0 \varepsilon_s}$$

Poznato F

Wednesday, May 18, 2022 10:09 AM

Procena vrednosti električnog polja koje konfinira elektrone na dopiranom heterospoju je F=5 MV/m. Odrediti energije prva tri nivoa u trougaonoj jami GaAs sa ovim nagibom. Da li se aproksimacija trougaone jame može primeniti na spoj GaAs/AlGaAs gde je visina barijere 0.3 eV

$\boxed{F = 5\,MV/m}$ $V(z) = eFz$ $\underbrace{\Delta E_C = 0{,}3eV}$



$$\mathcal{E}_n = C_n \mathcal{E}_0 = C_n \left[\frac{(eF\hbar)^2}{2m} \right]^{\frac{1}{3}}$$

$$m^{GaAs} = 0{,}067\, m_0$$

$$\mathcal{E}_0 = 0{,}0242\,eV = 24\,meV$$

$C_1 = 2{,}338$ $C_2 = 4{,}088$ $C_3 = 5{,}521$

$\mathcal{E}_1 = 57\,meV$ $\mathcal{E}_2 = 99\,meV$ $\mathcal{E}_3 = 134\,meV$

Poznato n2D

Wednesday, May 18, 2022 10:09 AM

Za zadatu koncentraciju 2DEG n2D = 3 10^15 1/m2, odredti električno polje koje generiše 2DEG i energije prva tri nivoa u trougaonoj GaAs jami sa ovim nagibom.

2DEG $\boxed{n_{2D} = 3 \cdot 10^{15}\ m^{-2}}$

$$F = \frac{e n_{2D}}{\epsilon_0 \epsilon_s} = 4{,}1125\ \frac{MV}{m}$$

$$\mathcal{E}_n = C_n \mathcal{E}_0 = C_n \left[\frac{(e\hbar)^2}{2m} \left(\frac{e n_{2D}}{\epsilon_0 \epsilon_s} \right)^2 \right]^{\frac{1}{3}}$$

$m = 0{,}067\, m_0$

$\epsilon_s = 13{,}2$

$\mathcal{E}_0 = 21\ meV$

$\mathcal{E}_1 = 50\ meV$, $\mathcal{E}_2 = 87\ meV$, $\mathcal{E}_3 = 117\ meV$

n2D max

Wednesday, May 18, 2022 10:11 AM

Odrediti maksimalnu koncentraciju 2DEG tako da je smao prva podzona popunjena na heterospoju modelovanom trougaonom jamom, čije je električno polje određeno koncentracijom n2D = 3 10^15 1/m2. (a) Pretpostaviti da se oblik potencijala ne menja sa porastom koncentracije. (b) Dozvoliti zavisnost elektronske strukture od koncentracije kroz Gausov zakon.

$n_{2D}^{(max)}$ САМО ПРВА ПОДЗОНА ПОПУЊЕНА

$$n_{2D} = 3 \cdot 10^{15}\ m^{-2}$$

(a) $$F = \frac{e n_{2D}}{\epsilon \epsilon_s} = 4{,}1\ MV/m$$

$$\varepsilon_0 = 21\ meV \qquad \varepsilon_1 = 50\ meV,\ \varepsilon_2 = 87\ meV$$

$$n_{2D} = \frac{m}{\pi \hbar^2} \sum_j (E_F - \varepsilon_j)\, \Theta(E_F - \varepsilon_j)$$

$$n_{2D}^{(max)} = \frac{m}{\pi \hbar^2}(\varepsilon_2 - \varepsilon_1) = 1{,}0416 \cdot 10^{16}\ m^{-2}$$

(b) $$n_{2D}^{(max)} = \frac{m}{\pi \hbar^2}(\varepsilon_2 - \varepsilon_1) = \frac{m}{\pi \hbar^2}(c_2 - c_1)\varepsilon_0$$

$$n_{2D}^{(max)} = \frac{m}{\pi \hbar^2}(c_2 - c_1)\left[\frac{\hbar^2}{2m}\left(\frac{e^2 n_{2D}^{(max)}}{\epsilon \epsilon_s}\right)^2\right]^{\frac{1}{3}}$$

$$\left(n_{2D}^{(max)}\right)^3 = \frac{m^3}{\pi^3 \hbar^6}(c_2 - c_1)^3 \frac{\hbar^2}{2m}\left(\frac{e^2 n_{2D}^{(max)}}{\epsilon_0 \epsilon_s}\right)^2 \frac{8}{8}$$

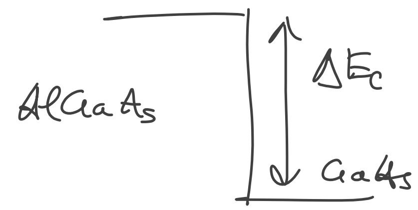
GaAs $$a_B = \frac{4\pi \epsilon_0 \epsilon_s \hbar^2}{m e^2} = 10{,}4\ nm$$

$$n_{2D}^{(max)} = \frac{8}{\pi}(c_2 - c_1)^3 \frac{m^2}{16\pi^2 \hbar^4}\left(\frac{e^2}{\epsilon_0 \epsilon_s}\right)^2 = \frac{8}{\pi}(c_2 - c_1)^3\left(\frac{m e^2}{4\pi \epsilon_0 \epsilon_s \hbar^2}\right)^2$$

$$n_{2D}^{(max)} = 1{,}3 \cdot 10^{17}\ m^{-2}$$

$$F = \frac{e n_{2D}^{(max)}}{\varepsilon_0 \varepsilon_s} = 172 \ \frac{MV}{m}$$

$$E_0 = 256 \ meV \qquad \Delta E_c = 0{,}3 \ eV$$

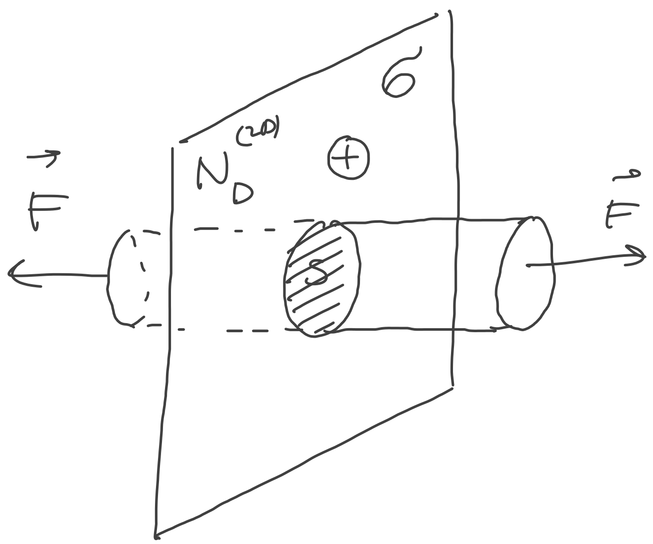


Simetrična trougana jama

Wednesday, May 18, 2022 10:41 AM

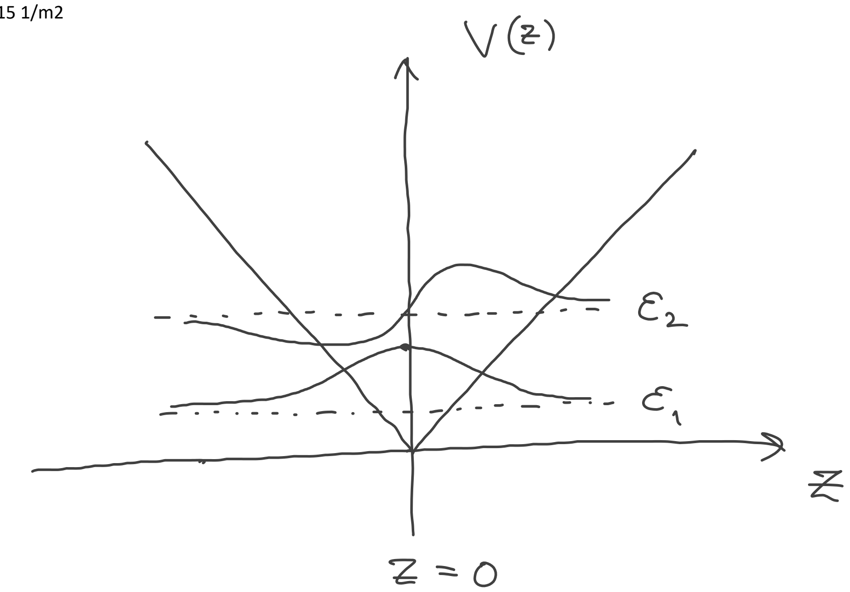
Odrediti energije prva dva nivoa u simetričnoj GaAs trougaonoj jami, V(z) = |eFz|, dobijenoj delta dopiranjem. Koncentracija donora po jedinici površine u tankom sloju je ND(2D) = 3 10^15 1/m2

$$V(z) = |eFz| \qquad N_D^{(2D)} = 3\cdot 10^{15}\ m^{-2}$$



$$\frac{\sigma S}{\epsilon_0\epsilon_s} = 2FS$$

$$F = \frac{\sigma}{2\epsilon_0\epsilon_s} = \frac{eN_D^{(2D)}}{2\epsilon_0\epsilon_s}$$



$$z>0, \quad V(z) = eFz \qquad \phi(z) = Ai\left[\frac{eFz-\varepsilon}{\varepsilon_0}\right] \qquad \varepsilon_n = c_n\varepsilon_0 = c_n\left[\frac{(e\hbar F)^2}{2m}\right]^{\frac{1}{3}}$$

$$F = 2{,}06\ MV/m \qquad \varepsilon_0 = 13{,}4\ meV$$

ЗА ПАРНА СТАЊА

$$\left.\frac{d\phi}{dz}\right|_{z=0} = 0 \Rightarrow Ai'\left(-\frac{\varepsilon}{\varepsilon_0}\right) = 0 \qquad -\frac{\varepsilon}{\varepsilon_0} = a_n'$$

$$a_1' = -1{,}019$$
$$a_2' = -3{,}248$$
$$a_3' = -4{,}820$$
$$a_4' = -6{,}163$$

$$\varepsilon_{2n-1} = -a_n'\varepsilon_0 \qquad \varepsilon_1 = 1{,}019\,\varepsilon_0 = 13{,}7\ meV$$

ЗА НЕПАРНА СТАЊА

$$\phi(z=0) = 0 \Rightarrow Ai\left(-\frac{\varepsilon}{\varepsilon_0}\right) = 0 \qquad -\frac{\varepsilon}{\varepsilon_0} = a_n$$

$$a_1 = -2{,}338$$

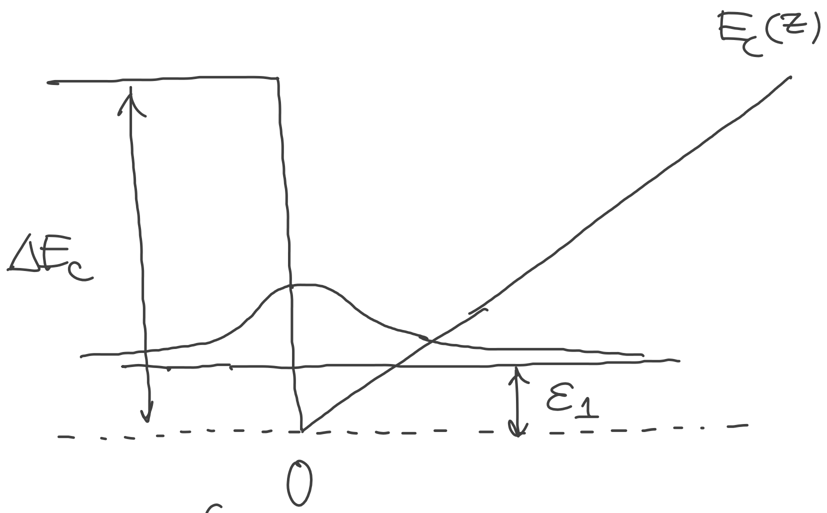
$$\varepsilon_{2n} = -a_n \varepsilon_0 \qquad \varepsilon_2 = 2{,}338 \cdot \varepsilon_0 = 31{,}3\,\text{meV}$$

$$a_2 = -4{,}088$$
$$a_3 = -5{,}521$$
$$a_4 = -6{,}787$$

Jama sa konačnom barijerom

Wednesday, May 25, 2022 12:31 AM

Proceniti energiju osnovnog stanja u trougaonoj kvantnoj jami sa konačnom barijerom: beskonačni potencijal u z=0, zamenjen je konačnim ofsetom u provodnoj zoni. Diskontinutet u prvodonoj zoni heterospoja GaAs/AlxGa1-xAs u Γ tački računa se kao 0.773x eV, a x = 0.3. Pretpostaviti da je energija osnovnog stanja bliska energiji osnovnog stanja u trougaonoj jami sa beskonačno visokom barijerom. Smatrati da je linearni potencijal kreiran 2DEG koncentracije 3 10^15 1/m2.



$$\mathcal{E}_1 = \mathcal{E}_1^{(0)} + \delta\mathcal{E}_1$$

$$n_{2D} = 3\cdot 10^{15}\ m^{-2}$$

$$F = \frac{e n_{2D}}{\varepsilon_0\varepsilon_s} = 4{,}1\ \frac{MV}{m}$$

$$\phi_1(z) = \begin{cases} A e^{\kappa z}, & z<0 \\ B\,Ai\left[\dfrac{eFz-\mathcal{E}_1}{\mathcal{E}_0}\right], & z>0 \end{cases}$$

$$\kappa = \sqrt{\frac{2m}{\hbar^2}\left(\Delta E_c - \mathcal{E}_1\right)}$$

$$\mathcal{E}_0 = \left[\frac{(eF\hbar)^2}{2m}\right]^{\frac{1}{3}}$$

$$V(z) = \begin{cases} \Delta E_c, & z<0 \\ eFz, & z>0 \end{cases}$$

$$\Delta E_c = 0{,}773\cdot 0{,}3 = 232\ meV$$

$$\phi_1(z=0^-) = \phi_1(z=0^+) \qquad A = B\,Ai\left(-\frac{\mathcal{E}_1}{\mathcal{E}_0}\right)$$

$$\phi_1'(z=0^-) = \phi_1'(z=0^+) \qquad \kappa A = B\,\frac{eF}{\mathcal{E}_0}Ai'\left(-\frac{\mathcal{E}_1}{\mathcal{E}_0}\right)$$

$$\kappa = \frac{eF}{\mathcal{E}_0}\,\frac{Ai'\left(-\frac{\mathcal{E}_1}{\mathcal{E}_0}\right)}{Ai\left(-\frac{\mathcal{E}_1}{\mathcal{E}_0}\right)} \qquad \mathcal{E}_1 = -a_1\mathcal{E}_0 + \delta\mathcal{E}_1 \qquad a_1 = -2{,}388$$

$$Ai\left(-\frac{\mathcal{E}_1}{\mathcal{E}_0}\right) = Ai\left(a_1 - \frac{\delta\mathcal{E}_1}{\mathcal{E}_0}\right) \approx \underbrace{Ai(a_1)}_{0} - \frac{\delta\mathcal{E}_1}{\mathcal{E}_0}Ai'(a_1)$$

$$Ai'\left(-\frac{\mathcal{E}_1}{\mathcal{E}_0}\right) = Ai'\left(a_1 - \frac{\delta\mathcal{E}_1}{\mathcal{E}_0}\right) \approx Ai'(a_1)$$

$$\kappa = \frac{eF}{\mathcal{E}_0}\,\frac{Ai'(a_1)}{-\frac{\delta\mathcal{E}_1}{\mathcal{E}_0}Ai'(a_1)} = -\frac{eF}{\delta\mathcal{E}_1} \qquad \boxed{\delta\mathcal{E}_1 = -\frac{eF}{\kappa}}$$

$$\kappa = \sqrt{\frac{2m}{\hbar^2}\left(\Delta E_c - \mathcal{E}_1\right)}$$

$$\downarrow$$

$$\delta\mathcal{E}_1 = -\frac{eF}{\kappa}$$

$$\downarrow$$

$$\mathcal{E}_1$$

1. $\varkappa = \sqrt{\frac{2m}{\hbar^2}\left(\Delta E_c - \varepsilon_1^{(0)}\right)}$ $\qquad \varepsilon_1^{(0)} = c_1 \varepsilon_0 = c_1 \left[\frac{(eF\hbar)^2}{2m}\right]^{\frac{1}{3}} = 50\ \text{meV}$

$\Delta E_c - \varepsilon_1^{(0)} = 182\ \text{meV}$

$\delta\varepsilon_1 = -7{,}3\ \text{meV}$

$\downarrow$

2. $\varepsilon_1 = \varepsilon_1^{(0)} + \delta\varepsilon_1$ $\qquad \Delta E_c - \left(\varepsilon_1^{(0)} + \delta\varepsilon_1\right) = 190\ \text{meV} \longrightarrow \varkappa$

$\delta\varepsilon_1 = -7{,}1\ \text{meV}$

$\boxed{\delta\varepsilon_1 \approx -7\ \text{meV}}$ $\qquad \boxed{\varepsilon_1 = \varepsilon_1^{(0)} + \delta\varepsilon_1 = 50 - 7 = 43\ \text{meV}}$

11.3

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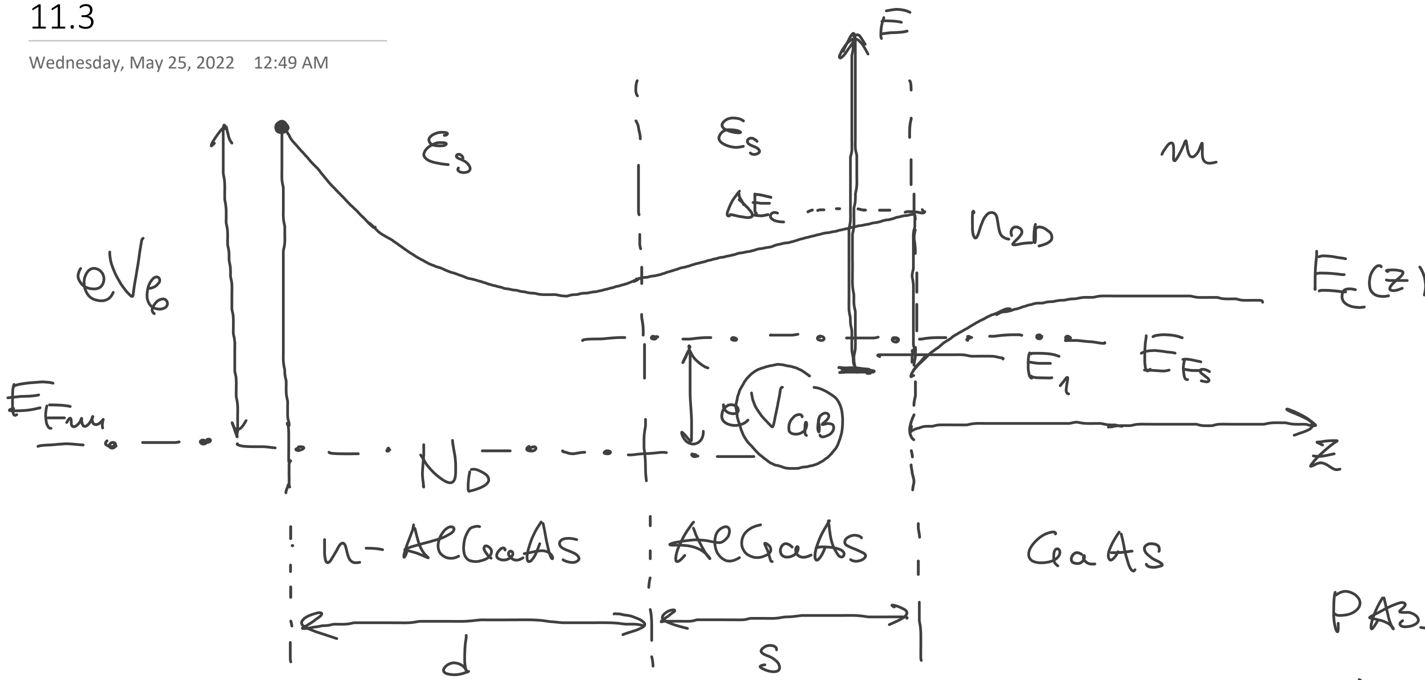


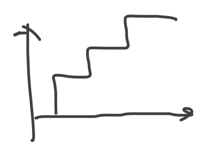
$$E_{Fm} = E_c(-(s+d)) - eV_b$$

$$E_c(z) = \Delta E_c - e\phi(z)$$

$$E_c(-(s+d)) = \Delta E_c - e\phi(-(s+d))$$

$$eV_{GB} = E_{Fs} - E_{Fm}$$

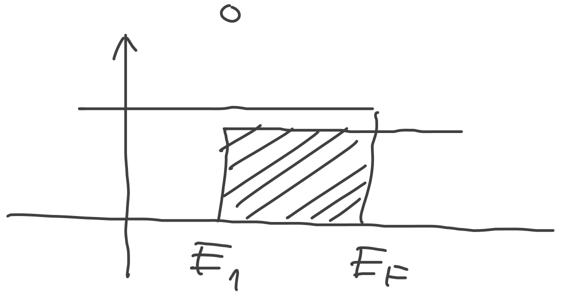
$$\rho_{2D} = \frac{m}{\pi\hbar^2}\sum_i \Theta(E-E_i)$$



$$n_{2D} = \int_0^{+\infty} \rho_1 f_{FD}(E)\,dE$$

$T = 0\,K \qquad f_{FD} = \Theta(E_F - E)$

$$n_{2D} = \frac{m}{\pi\hbar^2}\int_0^{+\infty}\Theta(E-E_1)\,\Theta(E_F-E)\,dE$$



$$n_{2D} = \frac{m}{\pi\hbar^2}(E_F - E_1)$$

РАЗДВОЈНИ СЛОЈ

$$\vec{F} = F\vec{e}_z \qquad F = \frac{e n_{2D}}{\varepsilon_0\varepsilon_s}$$

$$F = -\frac{d\phi}{dz}$$

$$\int_0^z d\phi = -F\int_0^z dz$$

$$\phi(z) - \cancel{\phi(0)} = -Fz \qquad \phi(0) = 0$$

$$\phi(z) = -Fz$$

$$-s < z < 0$$

$$E_{Fs} = E_1 + \frac{\hbar\pi}{m} n_{2D}$$

ДОПИРАНИ СЛОЈ

$$\frac{d^2\phi}{dz^2} = -\frac{\rho}{\varepsilon_0\varepsilon_s} = -\frac{eN_D}{\varepsilon_0\varepsilon_s}$$

$$\int_{-s}^{z} d\left(\frac{d\phi}{dz}\right) = -\frac{eN_D}{\varepsilon_0\varepsilon_s}\int_{-s}^{z} dz$$

$$\frac{d\phi}{dz} - \underbrace{\frac{d\phi}{dz}\bigg|_{-s}}_{-\frac{en_{2D}}{\varepsilon_0\varepsilon_s}} = -\frac{eN_D}{\varepsilon_0\varepsilon_s}(z+s)$$

$$\frac{d\phi}{dz} = -\frac{eN_D}{\varepsilon_0\varepsilon_s}(z+s) - \frac{en_{2D}}{\varepsilon_0\varepsilon_s}$$

$$\int_{-s}^{z} d\phi = -\frac{eN_D}{\varepsilon_0\varepsilon_s}\int_{-s}^{z}(z+s)dz - \frac{en_{2D}}{\varepsilon_0\varepsilon_s}\int_{-s}^{z} dz$$

$$\phi(z) - \phi(-s) = -\frac{eN_D}{2\varepsilon_0\varepsilon_s}(z+s)^2\bigg|_{-s}^{z} - \frac{en_{2D}}{\varepsilon_0\varepsilon_s}z - \frac{en_{2D}}{\varepsilon_0\varepsilon_s}s$$

$$\phi(z) - \cancel{\frac{en_{2D}}{\varepsilon_0\varepsilon_s}s} = -\frac{eN_D}{2\varepsilon_0\varepsilon_s}(z+s)^2 - \frac{en_{2D}}{\varepsilon_0\varepsilon_s}z - \cancel{\frac{en_{2D}}{\varepsilon_0\varepsilon_s}s}$$

$$\phi(z) = -\frac{eN_D}{\ldots}(z+s)^2 \quad \frac{en_{2D}}{\ldots}z \qquad -(s+d) < z < -s$$

$$E_{Fm} = E_c(-(s+d)) - eV_G = \Delta E_c - e\phi(-(s+d)) - eV_G$$

$$\phi(-(s+d)) = -\frac{eN_D}{2\varepsilon_0\varepsilon_s}(-d-s+s)^2 + \frac{e n_{2D}}{\varepsilon_0\varepsilon_s}(s+d) = \frac{e}{\varepsilon_0\varepsilon_s}\left[-\frac{N_D d^2}{2} + n_{2D}(s+d)\right]$$

$$E_{Fm} = \frac{e^2}{\varepsilon_0\varepsilon_s}\left[\frac{N_D d^2}{2} - n_{2D}(s+d)\right] + \Delta E_c - eV_G \qquad E_{Fs} = E_1 + \frac{\pi\hbar^2}{m} n_{2D}$$

$$-eV_{AB} = E_{Fm} - E_{Fs}$$

$$V_{AB} = V_G - \frac{\Delta E_c}{e} + \frac{e}{\varepsilon_0\varepsilon_s}\left[n_{2D}(s+d) - N_D\frac{d^2}{2}\right] + \frac{E_1}{e} + \underbrace{\frac{\pi\hbar^2}{em}} n_{2D}$$

$$\frac{\pi\hbar^2}{em}\,\frac{e}{\varepsilon_0\varepsilon_s}\,\frac{\varepsilon_0\varepsilon_s}{e}\,\frac{4}{4} = \frac{e}{\varepsilon_0\varepsilon_s}\,\frac{1}{4}\,\underbrace{\frac{4\pi\varepsilon_0\varepsilon_s\hbar^2}{e^2 m}}_{a_B} = \frac{e}{\varepsilon_0\varepsilon_s}\,\frac{a_B}{4}$$

$$V_{AB} = V_G + \frac{E_1 - \Delta E_c}{e} + \frac{e}{\varepsilon_0\varepsilon_s}\left[n_{2D}\left(s+d+\frac{a_B}{4}\right) - N_D\frac{d^2}{2}\right]$$

11.4

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$d = s = 20\,nm \quad \varepsilon_s = 12{,}2 \quad m = 0{,}067 m_0$

$eV_b = 0{,}57\,eV \quad \Delta E_c = 0{,}225\,V$

$N_D = 1{,}35 \cdot 10^{18}\,cm^{-3} \quad n_{2D} = 3 \cdot 10^{11}\,cm^{-2}$

$V_{GB} = 0{,}2\,V$

$$E_1 = eV_{GB} - eV_b + \Delta E_c - \frac{e^2}{\varepsilon_0 \varepsilon_s}\left[n_{2D}\left(s + d + \frac{a_B}{4}\right) - N_D \frac{d^2}{2}\right]$$

$$a_B = \frac{4\pi\varepsilon_0\varepsilon_s\hbar^2}{me^2} = 9{,}6358 \cdot 10^{-9}\,m = 9{,}64 \cdot 10^{-7}\,cm$$

$$e \quad \frac{e^{\leftarrow C}}{\underset{\underset{F/cm}{\uparrow}}{\varepsilon_0\varepsilon_s}} \underbrace{n_{2D}}_{cm^{-2}} \underbrace{\left(s + d + \frac{a_B}{4}\right)}_{cm} = 0{,}1887\,eV \qquad e \quad \frac{eN_D d^2}{2\varepsilon_0\varepsilon_s} = 0{,}4005\,eV$$

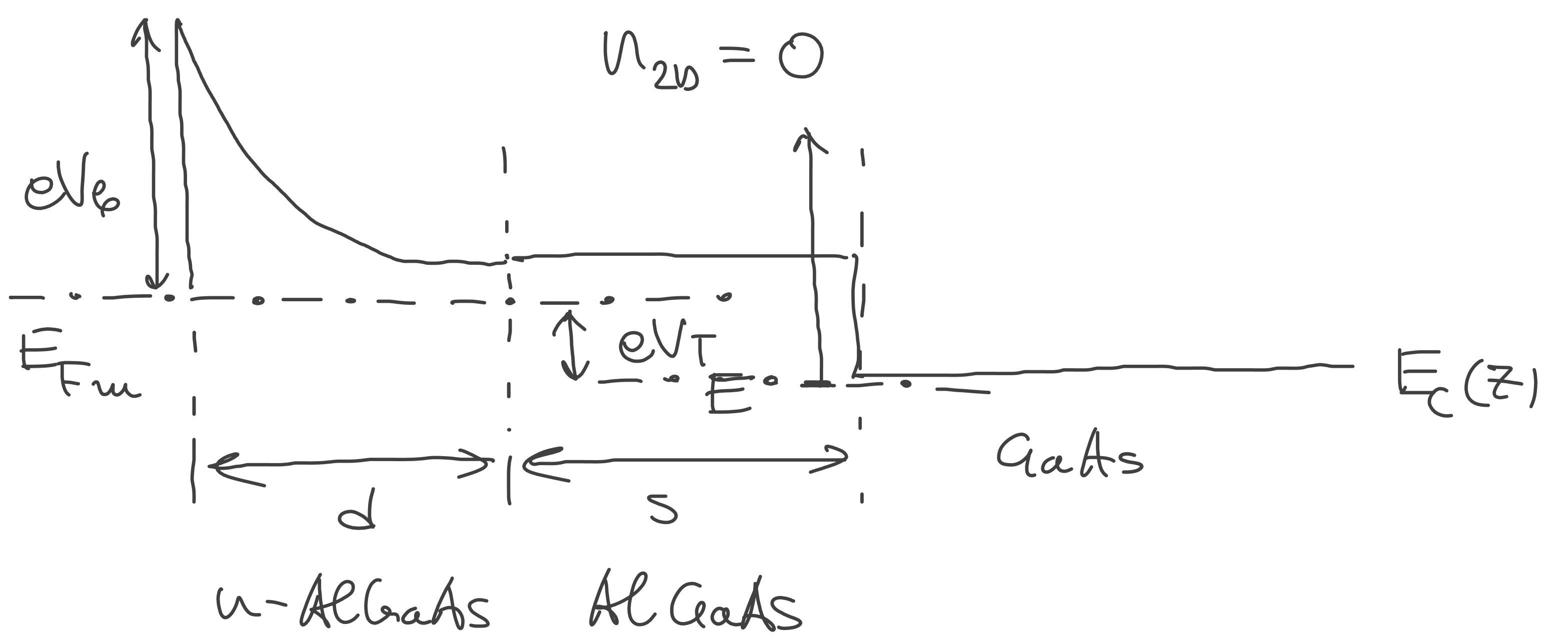
$$E_1 = 0{,}2 - 0{,}57 + 0{,}225 - 0{,}1887 + 0{,}4005 = 0{,}0668\,eV = 66{,}8\,meV$$

11.5

Wednesday, May 25, 2022 10:38 AM

$V_{GB} = V_T$

$n_{2D} = 0$



$E_1 = 0 \qquad E_{Fs} = E_1 + \frac{\pi \hbar^2}{2m} n_{2D} = 0$

$$V_{GB} = V_b + \frac{\cancel{E_1}^{\,0} - \Delta E_c}{e} + \frac{e}{\varepsilon_0 \varepsilon_S}\left[\cancel{n_{2D}}^{\,0}\left(s + d + \frac{a_B}{4}\right) - N_D \frac{d^2}{2}\right]$$

$$V_{GB} = V_b - \frac{\Delta E_c}{e} - \frac{e}{2\varepsilon_0 \varepsilon_S} N d^2 = -0,0555\,V = -55,5\,meV$$

Ugljenična nanotuba

Wednesday, May 25, 2022 10:49 AM

$$\vec{C}_h = n\vec{a}_1 + m\vec{a}_2 \qquad \vec{C}_h = 5\vec{a}_1 + 3\vec{a}_2$$

$$n\vec{a}_1 = n\frac{a\sqrt{3}}{2}\vec{e}_x + \frac{na}{2}\vec{e}_y$$

$$m\vec{a}_2 = m\frac{a\sqrt{3}}{2}\vec{e}_x - \frac{ma}{2}\vec{e}_y$$

$$|\vec{C}_h|^2 = \vec{C}_h \cdot \vec{C}_h = \left[(n+m)\frac{a\sqrt{3}}{2}\right]^2 + \left[(n-m)\frac{a}{2}\right]^2$$

$$|\vec{C}_h|^2 = n^2a^2 + m^2a^2 + nma^2$$

$$|\vec{C}_h| = a\sqrt{n^2+m^2+nm} \qquad n=5 \quad m=3$$

$$|\vec{C}_h| = 7a \qquad d_t = \frac{|\vec{C}_h|}{\pi} = 0{,}548\ nm$$

$$a = \sqrt{3}\,a_{cc} = 0{,}246\ nm$$

$$\vec{T} \perp \vec{C}_h \qquad \vec{T} = t_1\vec{a}_1 + t_2\vec{a}_2 \qquad n=5 \quad m=3$$

$$\vec{C}_h \cdot \vec{T} = 0 \Rightarrow \frac{t_1}{t_2}$$

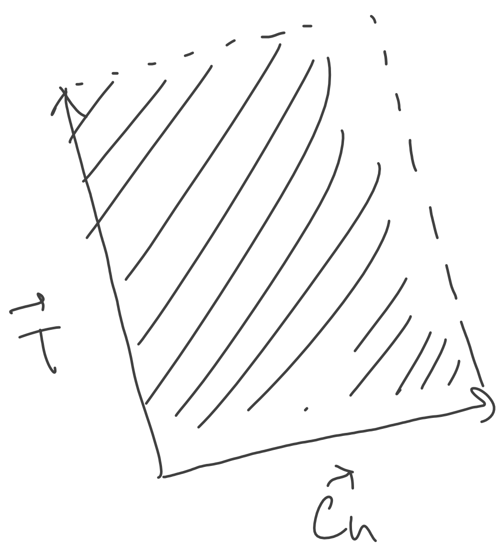
$$(t_1\vec{a}_1 + t_2\vec{a}_2)(n\vec{a}_1 + m\vec{a}_2) = nt_1a^2 + t_1m\underbrace{\vec{a}_1\cdot\vec{a}_2}_{\frac{a^2}{2}} + t_2n\underbrace{\vec{a}_2\cdot\vec{a}_1}_{\frac{a^2}{2}} + mt_2a^2 = 0$$

$$nt_1a^2 + mt_2a^2 + (t_1m + t_2n)\frac{a^2}{2} = 0$$

$$(2n+m)t_1 a^2 + (2m+n)t_2 a^2 = 0 \Rightarrow \frac{t_1}{t_2} = \frac{2m+n}{-(2n+m)}$$

$$t_1 = \frac{2m+n}{N_R} \qquad t_2 = -\frac{2n+m}{N_R} \qquad N_R = NZD\{(2m+n)(2n+m)\} = NZD\{11, 13\} = 1$$

$$t_1 = 11 \qquad t_2 = 13 \qquad \vec{T} = 11\vec{a}_1 + 13\vec{a}_2$$



$$N_c = \bar{N}\,|\vec{T}|\,|\vec{C_h}| \qquad |\vec{T}| = a\sqrt{t_1^2 + t_2^2 + t_1 t_2} = \sqrt{147}\,a$$

$$|\vec{C_h}| = a\sqrt{n^2 + m^2 + nm} = 7a$$

$6 \cdot \frac{1}{3}$

$$\bar{N} = \frac{2}{P} = \frac{2}{\frac{a^2\sqrt{3}}{2}} = \frac{4}{\sqrt{3}a^2} \qquad P_{⬡} = 6 \cdot \frac{a_{cc}^2\sqrt{3}}{4} = 6 \cdot \frac{a^2}{3} \cdot \frac{\sqrt{3}}{4} = \frac{a^2\sqrt{3}}{2}$$

$$N_c = \frac{4}{\cancel{\sqrt{3}}a^2} \cdot 7a \overset{7}{\cancel{\sqrt{147}}}\, a = 196$$